Long-time behavior of shape optimization solutions for the Navier-Stokes equations via a phase-field method

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Abstract

We investigate the asymptotic behavior of solutions to shape optimization problems governed by the time-dependent Navier–Stokes equations using a phase-field approach. The objective is to understand how the solutions behave as the time horizon $T \to \infty$, and in particular, how the minimizers of the corresponding cost functionals relate to those of the associated stationary problem. The optimization problems are formulated through phase-field relaxation, replacing sharp interfaces with diffuse ones governed by the Ginzburg–Landau energy. A porous media approximation is used to extend the governing equations to the full domain and enforce the impermeability condition. We provide the well-posedness and regularity results for both the time-dependent and stationary systems, as well as their associated linearized and adjoint equations. Our main result proves that the optimal value of the time-dependent functional converges to that of the stationary one, with a quantified convergence rate. Additionally, we show that a sequence of optimal phase-field functions corresponding to increasing time horizons admits a subsequence converging to a minimizer of the stationary problem.

Keywords: Navier-Stokes, phase-field method, porous media, optimization