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Wave-structure interactions

This course is devoted to the study of the interactions of waves at the surface of the ocean with a partially immersed structure (for instance a pontoon, a boat, a wave energy convertor). In the absence of an immersed structure, the motion of the waves are governed by the water waves (or free surface irrotational Euler) equations. However, for practical applications, simplified asymptotic models are used such as the linearized water waves equations, the nonlinear shallow water equations or the Boussinesq equations. We will describe in this course the mathematical issues that arise if one wants to use such approximations in the presence of a floating object.

Course I: **The linear approximation, Fritz John model and the Cummins equation**
In this course, we will analyse the linearization of the free surface equation with a floating object. This model was first studied by Fritz John and Cummins used it in the sixties to derive an integro-differential equation governing the motion of a floating object and still widely used by engineers. We will propose a rigorous analysis; despite the fact that the equations are linear, the analysis is non trivial because the fluid domain has corner singularities and that one needs to work in homogeneous spaces that are only semi-normed. (based on works with M. Ming and M. Paulsen)

Course II: **Wave structure interactions as partially constrained model**
We will describe here another approach to describe wave structure interactions, which is particularly convenient in shallow water. Its interest is that it allows one to take into account the nonlinear effects neglected in the previous effects and which are important for many applications (calibration of the mooring systems, evaluation of the wave energy potential...). The idea here is to write the system in terms of three variables: the surface elevation, the horizontal discharge, and the surface pressure. Different constraints on these quantities are imposed in the regions where the surface of the water is in contact with a floating object and the regions where it is in contact with the air. We will show how the wave structure interaction problem can be transformed into an exterior initial boundary value problem with non standard boundary conditions.

Course III: **Mathematical analysis of some one dimensional models**
We will study the initial boundary value problems resulting from the previous analysis for two waves model in horizontal dimension one: the nonlinear shallow water equations (NSW) and the Boussinesq equations (B). Since (NSW) are hyperbolic, the theory of initial boundary value problem (IBVP) is well developed. However, we will show that the dynamic of the contact line (point) between the object and the surface of the fluid is a free boundary problem of a new type, basically one derivative more singular that the shock stability problem. We will explain the importance of the Alinhac good unknown to solve such problems.
For (B), the presence of dispersive terms makes the analysis of the (IBVP) completely different and a new phenomenon arises at the dispersionless limit: the apparition of dispersive boundary layers. (based on works with D. Bresch and G. Métivier, T. Iguchi, and G. Beck).

Course IV: **Two-dimensional models**
We will comment here on the analysis of the (NSW) equations in horizontal dimension two. We explain the basics of hyperbolic (IBVP) and explain why the one we have to handle corresponds to the worst possible situation which remains completely open. We will however show that it is

possible to transform the problem so that it satisfies a new weak dissipativity property that we introduce, and that allows one to solve the problem. The dynamics of the contact line requires other developments (a new hidden regularity property, second order Alinhac good unknowns) that will be briefly addressed. (based on works with T. Iguchi)