Alexander Kurganov

Southern University of Science and Technology, China
alexander@sustech.edu.cn

Central-Upwind Schemes for Nonlinear Hyperbolic PDEs

I will first introduce general concepts of finite-volume (FV) methods for hyperbolic systems of conservation laws. These methods consists of three major steps: Reconstruction, Evolution, and Averaging.

Reconstruction means approximating the numerical solution, realized in terms of its cell averages, by piecewise polynomial functions. These reconstructions have to be conservative, high-order accurate, and non-oscillatory. To achieve the latter property, one needs to use nonlinear limiters, which i will briefly discuss.

Evolution is based on the integral form of conservation laws: this allows one to accurately handle discontinuous solutions, which typically arise even when the initial data are infinitely smooth. Depending on the way the evolution is conducted, FV schemes can be divided into two big classes: upwind and central. In upwind schemes, the solution is evolved using space-time control volumes (CVs), which are selected to align with the spatial mesh. Therefore, in order to implement upwind schemes, one has to (approximately) solve (generalized) Riemann problems, which might be a challenging task. On contrary, central schemes are based on different sets of space-time CVs, which are selected in such a way that the "Riemann fans" (waves generated at every cell interface) are contained inside the CVs: this allows one to avoid solving any Riemann problems. Hence, central schemes are simpler and typically more robust than their upwind counterparts.

Averaging requires a projection of the evolved solution onto the spatial mesh at the new time level, that is, upon completion of the evolution step. In upwind schemes, averaging is straightforward. In staggered (Lax-Friedrichs-type) central schemes, averaging is based on a very simple integration of piecewise polynomials. This averaging, however, introduces additional, often excessive numerical dissipation into the staggered central schemes. Central-upwind (CU) schemes are central schemes with a certain upwind feature. In CU schemes, CVs are selected based on an upwind information (one-sided local speeds of propagation), but the evolution is then conducted as in central schemes keeping CU schemes Riemann-problem-solver-free. On the other hand, averaging becomes crucial as the solution, which is evolved in time over a very "bad", strictly nonuniform mesh, has to be projected back onto the original mesh upon completion of every time step. This projection procedure may be conducted in several different ways leading to different CU schemes, which I will describe in details.

After introducing the CU schemes, I will speak on:

- Adaptive CU schemes;
- Well-balanced (WB) CU schemes for hyperbolic systems of balance laws;
- Path-conservative CU (PCCU) schemes for nonconservative hyperbolic systems;
- Flux globalization based WB PCCU schemes and their application to a variety of shallow water models.