A mathematical perspective on fluid-structure operators under Navier slip conditions

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Abstract

We analyze the motion of a rigid body immersed in an incompressible viscous fluid occupying the whole space, where the fluid evolves in the unbounded exterior of the body according to the Stokes equations with Navier-type slip conditions on the interface. The rigid body's translation and rotation obey Newtonian dynamics and are coupled to the fluid through boundary interactions.

While fluid-structure operators under no-slip conditions have been extensively studied in both bounded and unbounded domains, much less is known under slip boundary conditions-especially for exterior geometries. For bounded domains, operator-theoretic results such as analytic semigroups and maximal L^p-regularity have been established (e.g., Al Baba et al. [1]), but a comparable theory for the Stokes system with Navier slip in exterior domains remains underdeveloped.

In this work, we address this gap by combining bounded-domain fluid-structure theory with classical whole-space Stokes analysis. We construct an analytical framework suitable for the coupled system in an unbounded exterior domain with slip, and prove that the linearized operator generates an analytic semigroup and satisfies maximal L^p-regularity.

Moreover, we derive sharp L^r-to-L^q operator bounds:

$$\|u(t)\|_{L^q} \leq C\, t^{-\delta} \|u_0\|_{L^r},$$

with a decay exponent $\delta > 0$ depending on the indices and spatial dimension. These estimates capture the dissipative and smoothing

effects of the system and, although our present work focuses on the linearized dynamics, they are expected to play a key role in future analysis of nonlinear fluid-structure interactions in geometries exterior to a rigid body with Navier slip.

Keywords: fluid-rigid body interaction,fluid-structure operator, Navier slip boundary condition, Analyticity, L^q-L^r decay estimates,

References

[1] H. Al Baba, A. Ghosh, B. Muha, and Š. Nečasová, '*L^p-strong solution to fluid-rigid body interaction system with Navier slip boundary condition,* Journal of Elliptic and Parabolic Equations, Vol. 7, No. 2 (2021) ,439-489. doi:10.1007/s41808-021-00134-9