An attempt at energy equality for weak solutions to non-Newtonian viscous fluids in the shear thinning case

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Abstract

This presentation is based on a joint work with Francesca Crispo, Carlo Romano Grisanti and Paolo Maremonti.
In [1], we study the initial value problem of the power-law fluids in a spatially periodic domain:

$$\begin{aligned} &v_t - \nabla \cdot ((\mu + |\mathcal{D}v|^2)^{\frac{p-2}{2}} \mathcal{D}v) + v \cdot \nabla v + \nabla \pi_v = 0, \\ &\nabla \cdot v = 0, \text{ in } (0,T) \times \Omega, \\ &v(0,x) = v_0(x), \text{ on } \{0\} \times \Omega, \end{aligned} \tag{1}$$

where $\Omega := (0,L)^3$, $L \in (0,\infty)$, is a cube and we prescribe spatially periodic boundary conditions.

$$v|_{\Gamma_j} = v|_{\Gamma_{j+3}}, \ \nabla v|_{\Gamma_j} = \nabla v|_{\Gamma_{j+3}}, \ \ \pi_v|_{\Gamma_j} = \pi_v|_{\Gamma_{j+3}}, \tag{2}$$

with $\Gamma_j := \partial\Omega \cap \{x_j = 0\}$, $\Gamma_{j+3} := \partial\Omega \cap \{x_j = L\}$, $j = 1,2,3$.

The goal is the construction of a weak solution satisfying the energy equality.
Indeed, in the 2-dimensional case, it is known the existence of global strong solutions for $p > 1$ and these satisfy the energy equality.
In the 3-dimensional case, for $p \geq \frac{11}{5}$ it is well known the existence of global strong solution, so these satisfy the energy equality, but for $p < \frac{11}{5}$, global weak solutions to system (1) satisfy merely the energy *inequality.*

The approaches used, in the singular case, with $\mu > 0$, and $v_0 \in J^2(\Omega)$, allows to prove *an* energy equality which involves other quantities.
It is interesting to observe that the result is in complete agreement with what is known for the Navier-Stokes equations. Furthermore, in both cases, the additional dissipation, which measures the possible gap with the classical energy equality, is only expressed in terms of energy quantities.
Moreover, from a physical view point the energy relation would add a dissipative quantity which is not justifiable. Thus, the question arises of investigating the nature of these additional dissipation terms: they could be due to turbulence phenomena or to the weak regularity properties of the solution.

Keywords: power-law fluids, weak solutions, energy equality.

References

[1] Crispo F., Di Feola A.P., Grisanti C.R., Maremonti P.: An attempt at energy equality for weak solutions to non-Newtonian viscous fluids in the shear thinning case. *Forthcoming.*