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Course I: **The Coriolis force in turbulent flows, physical context, modelling and theory**
Flows subjected to solid body rotation are more easily studied in the rotating frame of reference. In this frame, both centrifugal and Coriolis forces affect the dynamics. In the incompressible case, only reviewed here for the sake of brevity, the centrifugal force can be incorporated in the pressure term, and eventually removed from consideration.
In many rotating flows, the remaining Coriolis force is weak, but has very subtle and complex effects. A brief review of these effects is offered for atmospheric and oceanic flows, at largest scales. Moreover, as an example of confined engineering flows, turbulence in the rotating channel flow is discussed, with a first discussion of its modelling. Looking more generally at multiscale and anisotropic approaches, it appears that the most difficult effect of the Coriolis force is the reduction of dimensionality. This effect cannot be directly accounted for in single-point closure methods [1,2], and requests two-point and even three-point description, closures and theories [2]. Accordingly, the multiscale anisotropy has to be disentangled in terms of directional anisotropy, connected to dimensionality, and polarization anisotropy. These two types of anisotropy are then discussed in both physical space and spectral one for typical flows of increasing complexity, quasi-static MHD, stably stratified, unstably stratified, and purely rotating. In the last case only, the purely linear analysis, sometime referred to as RDT (Rapid Distortion Theory), is poorly relevant, so that the interplay of linear effects with nonlinear ones is essential.
On the other hand, the theory of wave turbulence (WT) is well adapted for purely rotating flows [3]. The essentials of this theory are thereby presented, and its results for purely rotating flows are shown. Since the theory is an asymptotic approach, valid for very large Reynolds numbers and very low Rossby numbers, results from an anisotropic multimodal EDQNM closure, possibly matched with WT, and from DNS studies, are shown and compared, for intermediate ranges of Rossby and Reynolds number.

Course II: **Stably stratified and rotating flows, possible confinement**
(Julian Scott and Claude Cambon)
Linear theory (RDT) is partly relevant for stratified flows, with and without rotation. The most elaborated linear analysis of fluctuating velocity and density amounts to perform their normal mode expansion in 3D Fourier space. In addition to oscillating modes, which reflect inertia-gravity waves, a constant mode is found, as a non-propagating (NP) mode, connected to the quasi-geostrophic motion. The frequency of oscillation of the wavy mode, at a given orientation of the wave vector, is exactly given by the dispersion law of inertia-gravity (linear) waves. As an advantage, the normal modes decomposition remains valid for a fully nonlinear analysis (the modes are a complete basis), provided that their amplitudes become time-dependent and possibly coupled. As a drawback, WT is not directly applicable in the presence of a NP mode, since only a combination of wavy modes allows to drastically reduce the nonlinear transfer of kinetic, potential, energy, and crossed terms (fluxes), via wave dispersivity, as shown in purely rotating turbulence (end of the first talk.)
In addition to a wealth of DNS and LES, a multimodal anisotropic EDQNM approach, say EDQNM(2-3), was performed for stably stratified turbulence [4,2], in incorporating the NP mode. Results are shown and discussed. Such an approach can be generalized in principle to rotating stably stratified flows, but with formidable difficulties : the numerical resolution of EDQNM (2-3) is very demanding in calculation time, nothing to do with isotropic EDQNM, and realisability is not automatically fulfilled. A very promising new approach was proposed by Julian Scott, in keeping the essentials of wave turbulence theory only for the dynamics of the wave modes, and in

using a pseudo-spectral DNS, with particular adjustments. The new theoretical and numerical approach has four sounding results :

i) Decoupling of the NP mode from the wave modes is demonstrated (this was only suggested in former studies), firstly when the NP mode is initially small [5].

ii) This decoupling is generalized to the case of comparable wave and NP components [6]. Such a relevant contribution of the NP mode is incorporated in the WT dynamical equation of the wavy modes: A strong transfer mediated by mixed NP-waves components drastically affects the wavy modes and dominates their cascade.

iii) Analytical and numerical study in the weakly dispersive case, when the system vorticity 2Ω is close to the Brümt Wàisàla frequency N.

iv) Ignoring the wave modes, the QG (quasi-geostrophic) theory [8] is revisited and simplified, beyond the seminal Charnay's theory [9] and recent related studies.

Finally, effects of confinement were analyzed in a shearless rotating plane channel, with angular velocity perpendicular to solid walls [10,11]

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