On the ε-condition for dissipative solutions to the Naiver-Stokes equations

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Abstract

We consider local suitable weak solutions u to the Navier-Stokes equation in a three dimensional domain Ω. Such solutions satisfy a generalized local energy inequality and thus have similar properties as suitable weak solution introduced by V. Scheffer and Caffarelli-Kohn-Nirenberg. Furthermore, the existence and partial regularity of such solutions are known. More precisely, the following two ε-conditions have been established

$$r^{-2}\int_{Q(r)}|u|^3dxdt \le \varepsilon_0, \quad r^{-1}\|u\|^2_{L^\infty(t_0-r^2,t_0;L^2(B(r)))} \le \varepsilon_0$$

that guarantee the local regularity of u, where ε_0 stand for a positive absolute constant. In our talk we present our recent result that extends the above ε-condition by the following general version

$$r^{1-\frac{2}{p}-\frac{3}{q}}\|u\|_{L^p(t_0-r^2,t_0;L^q(B(r)))} \le \varepsilon_0 \quad \text{for all } \frac{3}{2} < q < 6, \quad \frac{2}{p}+\frac{3}{q} < 2.$$

The proof of this result is based on a new Caccioppoli-type inequality.