AEROACOUSTIC SIMULATION OF HUMAN PHONATION

J. Valášek [1], J. Horáček [2], P. Sváček [1]
[1] *CTU, Faculty of Mechanical Engineering, Dep. of Technical Mathematics, Karlovo náměstí 13, 121 35 Praha 2* [2] *Institute of Thermomechanics, CAS, Dolejškova 5, 182 00 Praha 8*

Summary

In this contribution the origin of human voice is investigated. Many theories try to explain how the voice is produced, see for example [Titze, R., Alipour, F.: The myoelastic aerodynamic theory of phonation, 2006], but until now it has been fully understood. The human phonation represents a coupled problem composed of three different physical fields – the deformation of the vocal folds (elastic body), the complex fluid flow and the acoustics together with all mutual couplings. It is sometimes referred as fluid-structure-acoustic interaction problem. Due to very complicated accessibility of human airway the numerical simulation plays irreplaceable role.

The critical part of whole simulation is the reliable solution of fluid-structure interaction (FSI) problem. To include the effects of time-dependent computational domain the Arbitrary Lagrangian-Eulerian (ALE) method was used. The fluid flow is modelled by incompressible Navier-Stokes equations in ALE formulation and structure is described by linear elasticity theory. The acoustic part is here solved with the aid of acoustic analogies because direct computation of acoustics within compressible fluid flow simulation would have to face too many problems. One from many is the magnitude disparity of acoustic pressure and overall hydrodynamic pressure, see [1]. Therefore the hybrid approach, when first FSI problem is solved and later according to the chosen acoustic analogy the sound propagation is computed, can benefit from problem-specific solvers and substantial lower computation demand, see [2]. The Lighthill acoustic analogy and acoustic wave equation (AWE) are introduced here.

The all three mentioned problems are solved by finite element method (FEM). The SUPG stabilization and mini-element in flow solver was implemented. Finally, the propagation of acoustic pressure through model of human vocal tract based on data from FSI simulation was computed. The frequency spectra of signal in front of mouth was obtained by the Fourier transform and compared.

Fluid structure interaction

Elastic body. Deformation $\mathbf{u}(x,t)$ is given by

$$\rho^s \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \tau^s_{ij}}{\partial x_j} = f^s_i \quad \text{in } \Omega^s \times [0,\mathrm{T}], \tag{1}$$

where ρ^s is density and $\mathbf{f}^s$ volume force. The Cauchy stress tensor τ^s_{ij} can be expressed by Hooke law with help of Lame coefficients λ^s, μ^s and tensor of small displacement e^s_{ij} as $\tau^s_{ij} = \lambda^s(\mathrm{div}\,\mathbf{u})\,\delta_{ij} + 2\mu^s e^s_{ij}(\mathbf{u})$.

Fluid flow. The ALE method is used for purpose to include effects of fluid domain change in time. The key of the method is to construct diffeomorphism A_t from reference state onto deformed state in any time instant t, i.e. $A_t : \overline{\Omega}^f_{ref} \longrightarrow \overline{\Omega}^f_t$.

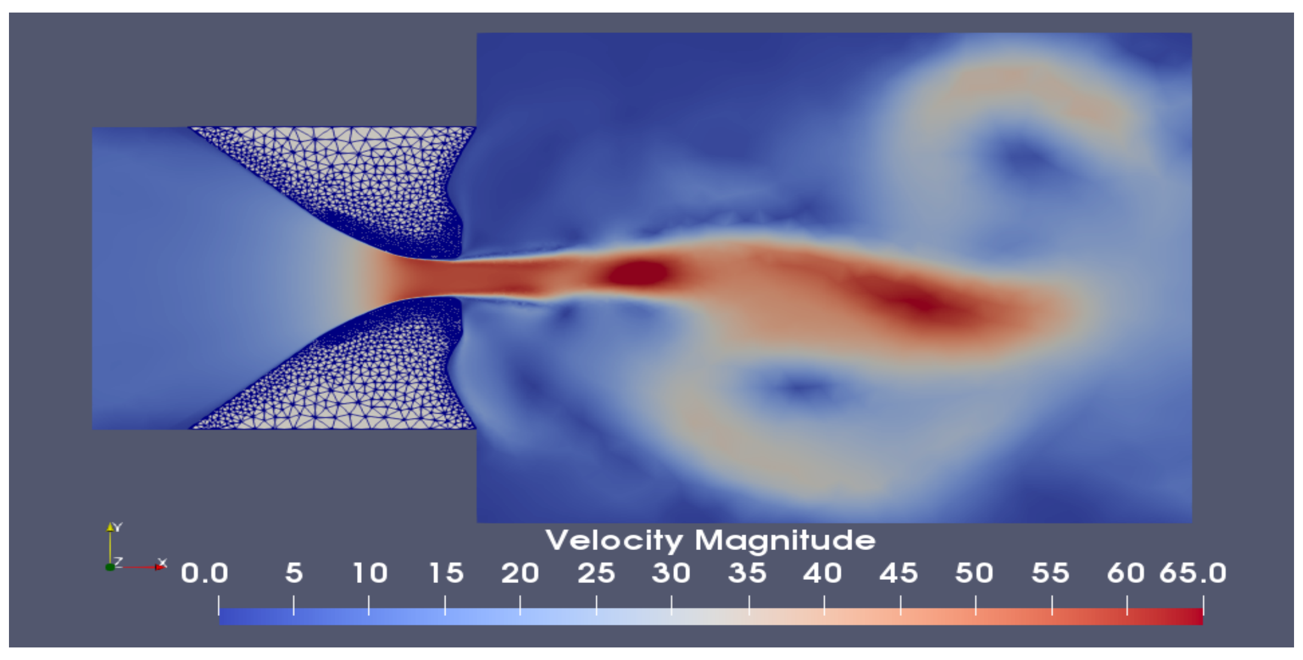
Than Navier-Stokes equations in ALE form are

$$\frac{D^A\mathbf{v}}{Dt} + ((\mathbf{v}-\mathbf{w}_D)\cdot\nabla)\mathbf{v} - \nu^f\Delta\mathbf{v} + \nabla p = \mathbf{g}^f, \qquad \mathrm{div}\,\mathbf{v} = 0 \qquad \text{in } \Omega^f_t, \tag{2}$$

where ν^f is kinematic viscosity, $\mathbf{w}_D$ is the domain deformation velocity and $\frac{D^A}{Dt}$ denotes the ALE derivative, see e.g. [3].

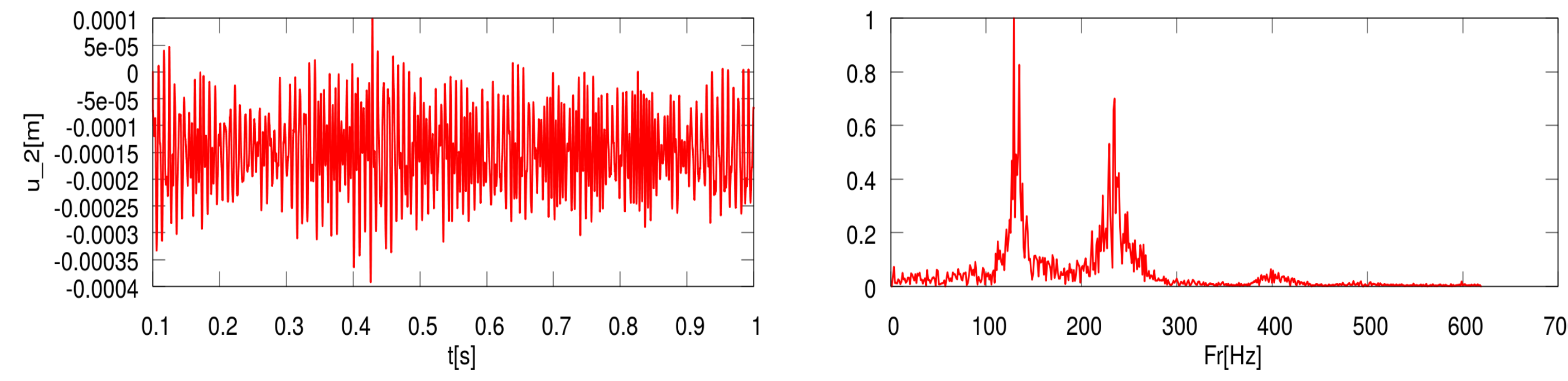
Equations (1) and (2) are supplied with suitable initial and boundary conditions, specially on the common interface Γ_{W_t} the dynamic and kinematic boundary conditions are prescribed.

The FSI problem was numerically approximated by FEM and partitioned scheme approach was used, see [4]. The typical distribution of **fluid velocity** in time instant $t = 0.0355$ s



.

Stored **displacement** of the top point from bottom vocal fold in y-direction is shown left. Its **Fourier transform** is depicted right.



Aeroacoustic analogies

Lighthill acoustic analogy. The propagation of pressure fluctuation p' is described by inhomogeneous wave equation

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x_i^2} = \frac{\partial^2 L_{ij}}{\partial x_i \partial x_j}, \tag{3}$$

where Lighthill tensor is $L_{ij} = \rho^f v_i v_j + ((p') - c_0^2(\rho'^f))\delta_{ij} - \tau^f_{ij}$ determines the sound sources. Without considering of heat conduction it can be approximated as $L_{ij} \approx \rho^f v_i v_j$, see [5]. The drawback of Lighthill analogy is bias of acoustic results in near field, see [1].

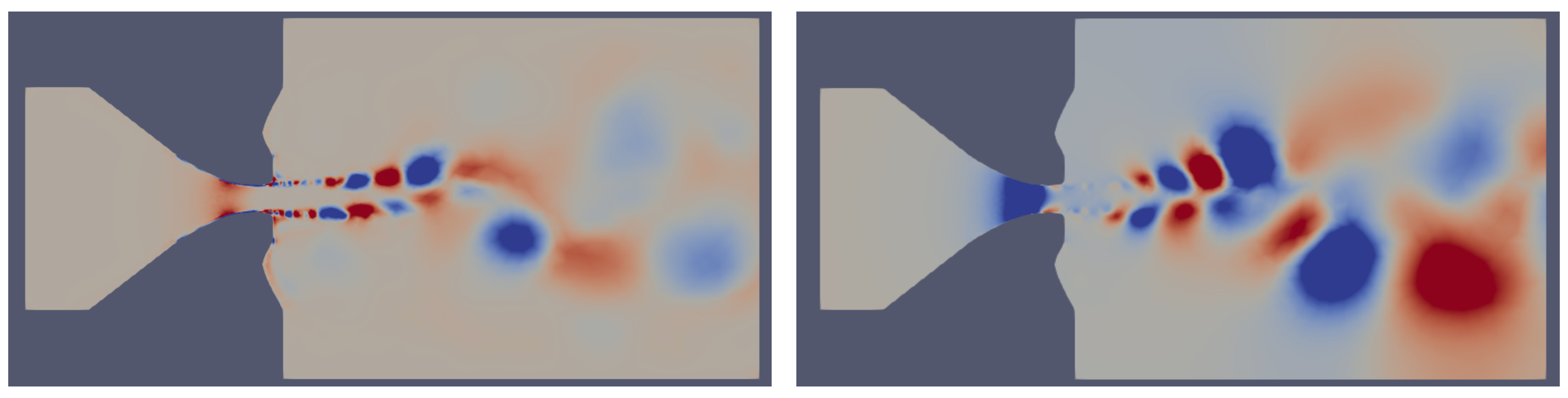
AWE analogy. The AWE analogy is formulated directly for acoustic pressure p^a

$$\frac{1}{c_0^2}\frac{\partial^2 p^a}{\partial t^2} - \Delta p^a = -\frac{1}{\rho^f_0 c_0^2}\frac{\partial^2 p_{ic}}{\partial t^2}. \tag{4}$$

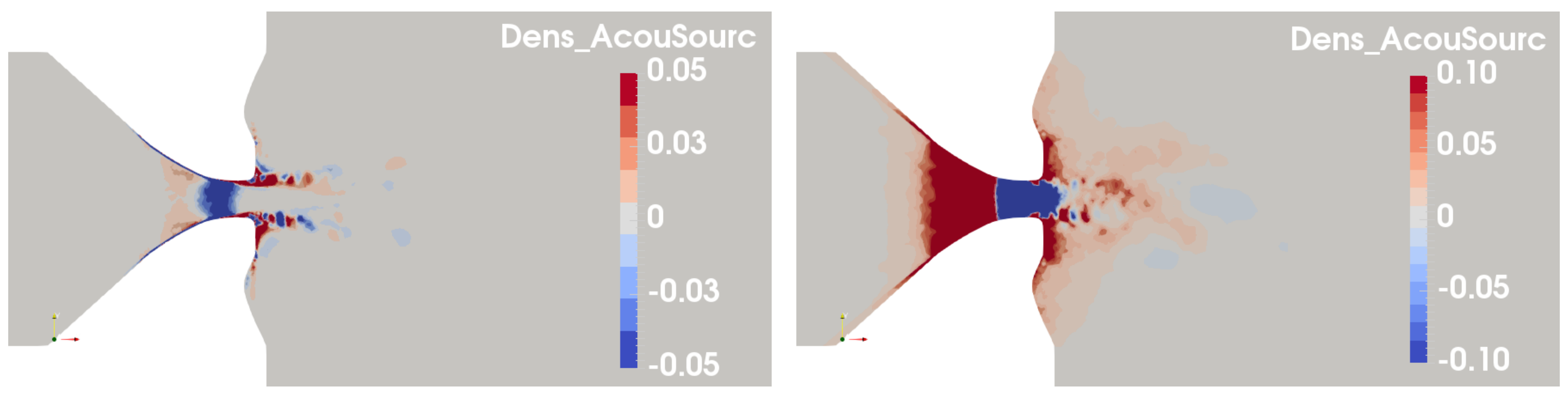
It is derived based on splitting pressure into mean and fluctuating non-acoustic and acoustic parts, i.e. $p = \overline{p} + p_{ic} + p_a$ and on the assumption of irrotational acoustic field, see [6]. Its advantage is acoustic source given by time derivative not space derivative.

Numerical results

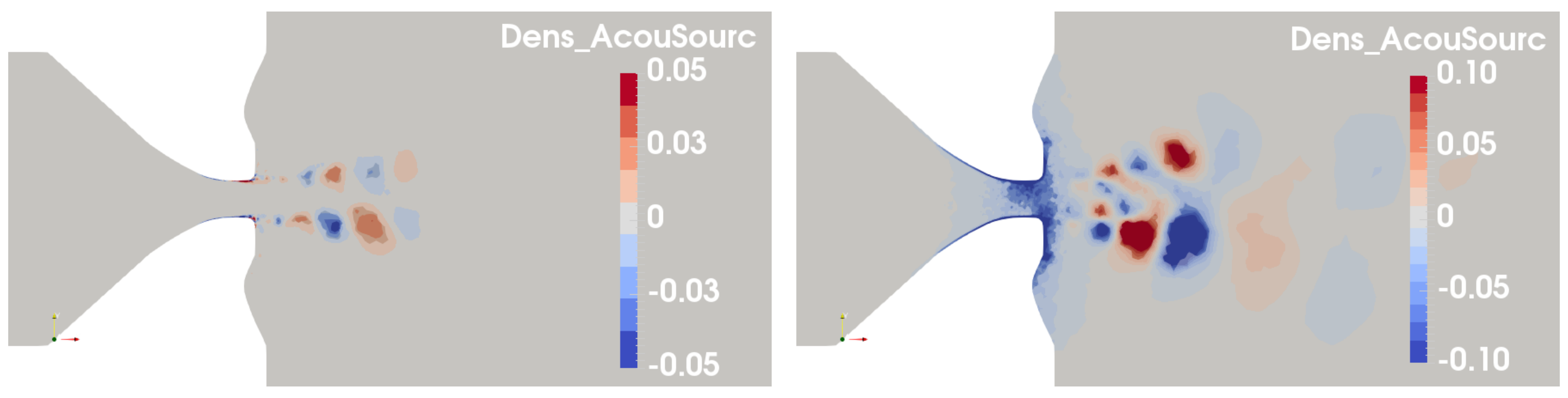
Sound sources. The computed sound source densities according to Lighthill analogy at time instant $t = 0.025$ s are shown left, respectively for AWE analogy right.



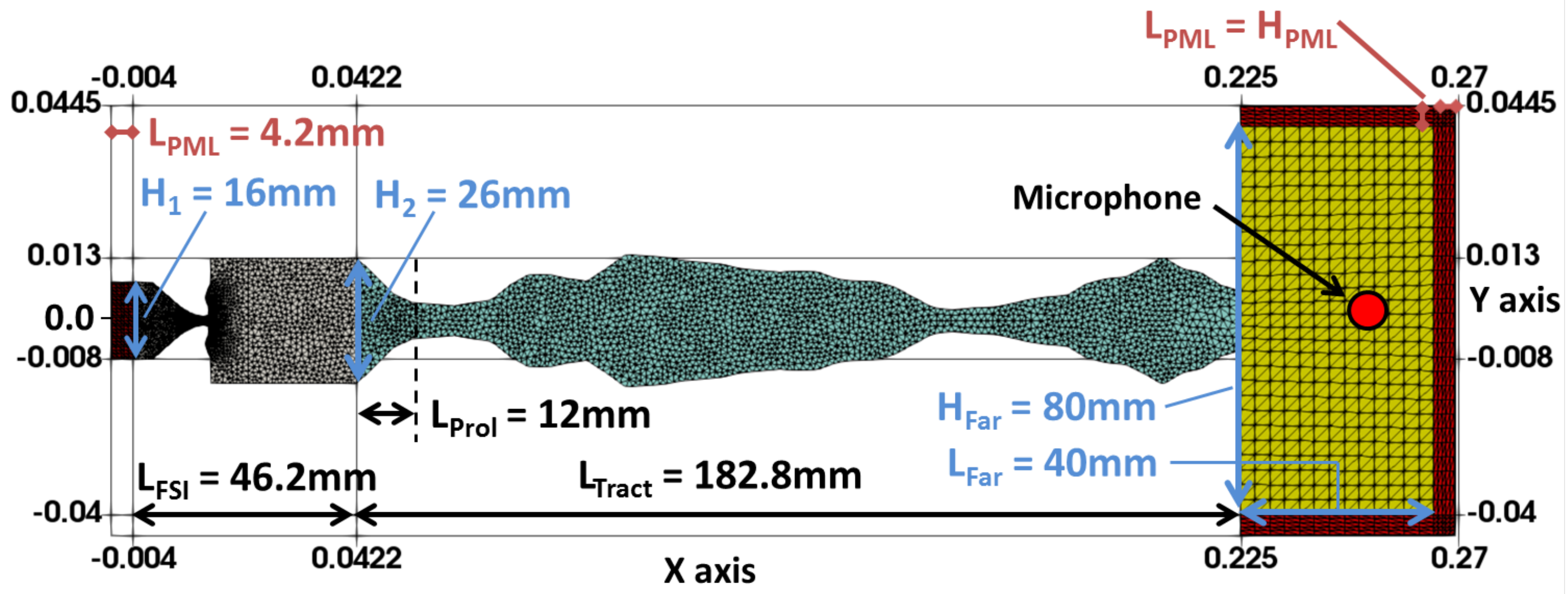
The structure of sound sources at frequency 232 Hz is displayed for Lighthill analogy left and for AWE right.



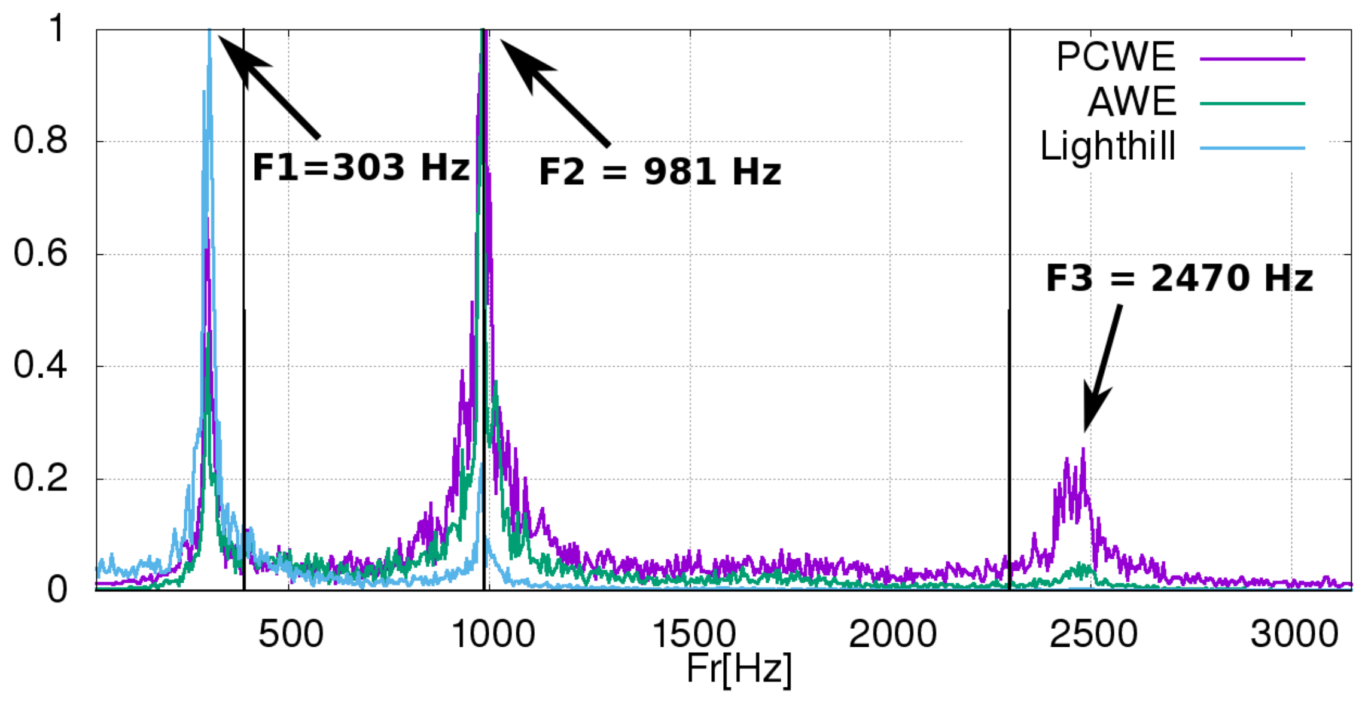
The sound sources of frequency 2486 Hz are for Lighthill analogy left and for AWE right.



Sound spectrum at mouth position. The model of vocal tract for vowel [u:] motivated by [7] is sketched below with highlighted microphone position.



The frequency spectra obtained by Fourier transform of acoustic signal at microphone position for Lighthill analogy, AWE analogy and PCWE analogy, see [1], are compared with measured formants of natural speech, see [7], represented in Figure by black lines.



Conclusion. .

- FSAI problem was mathematically formulated.
- All subproblems were numerically solved by FEM and implemented.
- Sound sources based on FSI results computed, interpolated and analyzed.
- Frequency spectra of three different acoustic analogies were compared with formants of vowel [u:] from article [7].

Acknowledgment. The financial support was provided by *Grant No. GA 19-04477S* of Czech Science Foundation and by *Grant No. SGS19/154/OHK2/3T/12* of the CTU in Prague. Authors thanks for possibility to use scientific FE library *CFS++*.

References. .

[1] M. Feistauer, P. Sváček, J. Horáček, in *Fluid-structure Interaction and Biomedical Applications*, ed. by T. Bodnár, G.P. Galdi, S. Nečasová, Birkhauser, 2014.

[2] A. Hüppe, M. Kaltenbacher, Journal of Computational Acoustics, **20**, 2012.

[3] M. Kaltenbacher, *Numerical simulation of mechatronic sensors and actuators*, 2015.

[4] M.J. Lighthill, in *Proceedings of the Royal Society of London*, **211**, 1952.

[5] P. Šidlof, S. Zörner, A. Hüppe, Biomechanics and Modeling in Mechanobiology, 2014.

[6] B. Story, I. Titze, E. Hoffman, Journal of the Acoustical Society of America, 1996.

[7] J. Valášek, M. Kaltenbacher, P. Sváček, in *Topical problems of fluid mechanics 2017*.