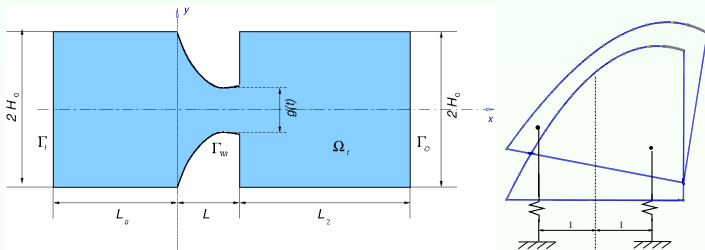


# On mathematical modelling of human phonation process: Finite element approximation of flow induced vocal folds vibrations

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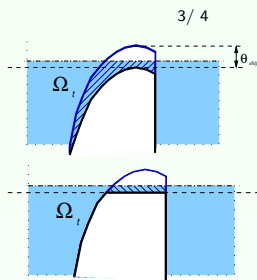
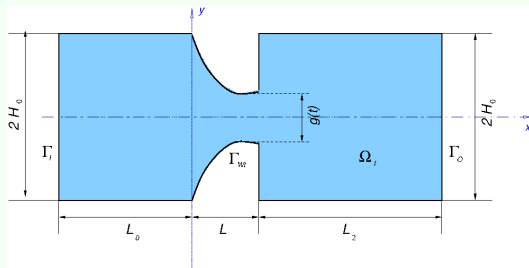
## Mathematical model in $\Omega_t$ (ALE method)

$$\frac{D^A \mathbf{u}}{Dt} + (\mathbf{u} - \mathbf{w}_g) \cdot \nabla \mathbf{u} + \nabla p - \nu \Delta \mathbf{u} = 0, \quad \nabla \cdot \mathbf{u} = 0$$

## Structure model :

$$\mathbb{M} \begin{pmatrix} \ddot{w}_1 \\ \ddot{w}_2 \end{pmatrix} + \mathbb{B} \begin{pmatrix} \dot{w}_1 \\ \dot{w}_2 \end{pmatrix} + \mathbb{K} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} + \begin{pmatrix} F_1^S \\ F_2^S \end{pmatrix} = 0$$

# How to treat contact?



## Modifications to treat the contact:

- Structural model - Hertz impact forces.

$$F_H = k_H \delta^{3/2} (1 + b_H \dot{\delta})$$

- Flow model - inlet/outlet b.c!

Heywood, Rannacher, Turek, 1996, Bruneau, Fabrie, 1996, Braack, Mucha 2014, Svacek, Horacek 2017

$$-\nu \frac{\partial \mathbf{u}}{\partial \mathbf{n}} + \frac{1}{2} (\mathbf{u} \cdot \mathbf{n})^- \mathbf{u} + (p - p_{ref}) \mathbf{n} = \frac{1}{\epsilon} (\mathbf{u} - \mathbf{u}_{ref})$$

- Flow model - modify geometry and governing equations!

$$\frac{D^A \mathbf{u}}{Dt} + (\mathbf{u} - \mathbf{w}_g) \cdot \nabla \mathbf{u} + \alpha \mathbf{u} + \nabla p - \nu \Delta \mathbf{u} = 0, \quad \nabla \cdot \mathbf{u} = 0$$

# Numerical results

Flow patterns, fictitious porous media domain (in red) during the glottis closure phase:

