On asymptotic stability of Boussinesq equations

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Abstract

We consider the motion of viscous incompressible fluid under the action of gravitation/buoyancy modeled by the Boussinesq equations in the absence of thermal conduction,

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} - \Delta \mathbf{u} + \nabla p = \theta \mathbf{e}_2, \ \mathrm{div}\mathbf{u} = 0,$$

$$\partial_t \theta + \mathbf{u} \cdot \nabla \theta = 0.$$

This is a simple nonlinear elliptic-parabolic-hyperbolic coupled system. Consider the specific stationary solution

$$\mathbf{u}_s = \mathbf{0}, \ \theta_s(y) = y, \ p_s(y) = \frac{1}{2} y^2, \ y \in (0, 1)$$

in the two dimensional domain $\mathbb{R} \times (0, 1)$. The perturbed system around this solution reads as

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} - \Delta \mathbf{u} + \nabla q = \vartheta \mathbf{e}_2, \ \mathrm{div}\mathbf{u} = 0,$$

$$\partial_t \vartheta + \mathbf{u} \cdot \nabla \vartheta = -u_2.$$

Under *suitable boundary conditions* we show global existence and decay rates of solutions to the perturbed system with small initial data. This in turn implies asymptotic stability of the specific stationary solution.

Some possible extension and unsolved problems will also be discussed. This is a joint work with Lihua Dong.