Design-through-Analysis

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Abstract

Isogeometric Analysis (IGA) [1] is a recently introduced extension of the Finite Element Method (FEM) that aims at bringing the disciplines of computer-aided geometric design modeling, simulation-based analysis, and design optimization closer together. It is based on the idea of using the same mathematical tools, i.e. multi-variate (adaptive) B-Splines and Non-Uniform Rational B-Splines (NURBS) to represent solutions and geometries. This brings new scientific challenges, e.g., the 'meshing problem' now becomes the problem of generating analysis-suitable domain parametrizations, but also opens new opportunities for computer-aided design-through-analysis workflows.

This course will start with an introduction into the fundamentals of IGA, which includes an overview of the basics of spline technologies and a discussion of efficient assembly and solution techniques. In subsequent sessions we will investigate strategies for generating analysis-suitable domain parametrizations, and IGA applications in design optimization. The following topics with be discussed:

(1) IGA fundamentals (part 1).

a) Introduction to multi-variate B-splines and NURBS

b) Geometry modeling and PDE analysis with splines

c) Multi-patch coupling and adaptive spline technologies

(2) IGA fundamentals (part 2).

a) Efficient assembly of system matrices and vectors

b) Efficient solution techniques for IGA discretizations

(3) Analysis-suitable parametrizations.

a) Parametrization techniques for surfaces and volumes

b) PDE-based parametrization techniques for complex geometries

(4) Gradient-based design optimization with IGA.

a) Introduction to gradient-based design optimization

b) Algorithmic differentiation and computational aspects

c) Selected Applications

Reading material:

A general introduction into this topic by the founders of IGA can be found in their textbook [2] and the original research article [1]. An alternative introduction to IGA tailored to finite element users is given in [3]. Two prominent adaptive spline technologies are THB-splines [4] and LR-splines [5, 6]. A good overview of state-of-the-art assembly techniques for IGA discretizations can be found in the publications [7, 8, 9, 10] and the references therein. Some milestones of IGA solvers are described in [11, 12, 13]. For a comprehensive overview of state-of-the-art parametrization techniques we refer the interested reader to the PhD thesis by Hinz [14] and the references therein. Gradient-based design optimization and algorithmic differentiation are introduced in [15] and [16], respectively.

It is not recommended nor expected to study all material in advance. A basic understanding of the FEM and, if time permits, some preparation of basic spline technologies is sufficient for following the course.

References

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