Mathematical Theory of Fluid/Flow-Structure Interactions. Analysis and Control

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1. General considerations. These Lecture are devoted to the study of PDE-models which describe complex dynamical systems occurring in modern scientific applications such as fluid and flow-structure interactions. These are coupled PDE-systems where coupling occurs at an interface that separates two physical domains on which two different dynamical environments evolve (e.g. solid and fluid; or solid and wave). It is precisely the influence of the interface that plays a predominant role in determining the resulting dynamical properties of the overall coupled system. Its impact is reflected in the underlying technical analysis of the entire dynamical unit. The interface is the region where properties of a single dynamical component of one medium propagate onto the other medium, possibly changing drastically the properties of the second dynamical environment. Sometimes these changes are deleterious, even catastrophic; sometimes they are sought-after targets, very beneficial and desirable for the overall coupled system.

Illustrative examples of the first type include: (i) elastic properties of an artery that effect the blood-flow within it, thus causing high blood pressure; (ii) wind-induced vibrations of an oscillating structure that eventually determine fatigue failure (as in the collapse of the Tacoma Narrows Bridge); (iii) dangerous vibrations of an airfoil due to strong headwinds, which may lead to loss of stability in flight; etc. Such examples are ubiquitous. For these, one would like, first, to determine the qualitative behavior of the uncontrolled solutions of the PDE-coupled system and, next, design suitable controllers - most desirably in feedback form and *finite dimensional* - capable to suppress or prevent instability and/or catastrophic regimes. The phenomenon of 'bad-outcome suppression' by feedback controllers is what is generally referred to as 'feedback stabilization' problem. Canonical illustrations may be: asymptotic noise suppression in an acoustic chamber or aircraft cockpit or cabin; asymptotic turbulence suppression of a fluid, etc .

In recent years serious advances have been made in the mathematical study of fluid-structure and flow-structure interactions. As a result, we now have a reasonably good understanding of issues related to well-posedness of these dynamics—both local and global, as well as their long time behavior and culminating with a *finite dimensional description* of asymptotic dynamics. The latter being amenable to finite dimensional control. One of the fundamental challenges is obtaining a good understanding of the propagation of either energy decay, or of the regularity properties, from one component of the coupled system to the other via interaction at the interface. Since the interface involves boundary traces of the respective PDE solutions, new developments in (hidden/sharp) regularity of traces play a key role, typically via microlocal analysis.

2. Fluid-structure interaction, the structure–modeled by a hyperbolic system of dynamic elasticity—is immersed in a fluid—modeled by the parabolic Navier-Stokes equations—with matching velocities and stresses at the interface between the two respective domains. Though fluid/flow-structure interactions are ubiquitous in nature, till very recently their study has been primarily conducted through numerical/engineering approaches and has appeared in the literature of the respective engineering and scientific communities. In contrast, only over the past ten years or so has this topic attracted mathematical studies of some of its foundational properties. This includes global theories for existence of solutions to quasilinear coupled systems with a "mis-match" of regularity along with new unique continuation properties associated with controllability and stabilization. of the dynamics.

3. Flow structure interaction—a dynamic plate (or shell) interacts with a surrounding air-flow. The dynamics of the flow is hyperbolic-like modeled by compressible, irrotational Euler equation linearized around unstable flow profile. The interaction is via aeroelastic potential over the whole structure[plate or shell] as well as on the boundary. Here, well-posedness of finite energy solutions is first established. This result will then be followed by a construction of a compact global attractor that captures the long-time behavior of the nonlinear structural dynamics. The originally rough and oscillatory dynamics are shown to stabilize to a smooth and finite dimensional set (in some cases, at exponential rate). The ultimate dynamics may exhibit a chaotic behavior. Conditions under which solutions are shown to stabilize to the equilibria set will be discussed. Since the frame is often moving (with the velocity of the fluid or flow), this leads to delicate free boundary problems.

These lecture notes are intended to achieve a few goals: (i) to describe, in a "user-friendly" way, recent advances in the appropriate corresponding areas, and (ii) to indicate attractive directions for future research, including several open problems.

4. Plan of the lectures:

- Lecture 1:
  - Introduction to coupled systems with an interface. Static and moving configurations. Examples of fluid and flow structure interactions.
  - Generation of dynamical systems associated with the interface systems. Weak and strong solutions.
- Lecture 2:
  - Flow-Structure Interaction-Generation of a nonlinear semigroup.
  - Fluid-Structure interaction with moving domain. Global in time solvability.
- Lecture 3
  - Abstract theory of long time behavior in dynamical systems. Nondissipative dynamics and quasistable systems.
- Lecture 4:
  - Long time behavior and feedback control of flow structure interaction.
    - ∗ Convergence to equilibria.
    - ∗ Finite dimensional asymptotic behavior and its control.
  - Conclusions and Open Problems.

Suggested reading

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