On the stabilization problem by feedback control for some hydrodynamic type systems

Andrei Fursikov
Lomonosov MSU, Moscow, Russia
fursikov@gmail.com

Abstract

The main hydrodynamic system that we study here is three-dimensional (3D) Navier-Stokes equations of viscous incompressible fluid. First of all after short introduction we will recall some old result on the nonlocal exact controllability of 3D Navier-Stokes system in the class of smooth solutions that is basic for our investigation.

The second lecture will de devoted to study the local stabilization problem for Navier-Stokes system by feedback control.

In the third lecture we will choose some as simple as possible formulation of nonlocal stabilization problem for 3D Navier-Stokes system that nevertheless contains some serious difficulties. We reduce this problem to equivalent one for Helmholtz system. The nonlinear term of Helmhotz system is the sum of so called normal and tangential nonlinearities. On the first step of our investigation we take away tangential nonlinearity in Helmholtz system and obtain so called Normal Parabolic System (NPS). The third lecture will be devoted to study properties of NPS including its nonlocal stabilization.

After that we have to put back tangential nonlinearity to NPS and continue investigation of Helmholtz system. We will do it but only in the case of more simple Burgers type equation, i.e. we will do only the first step to this direction. The forth lecture will be devoted to this topic.

(1) Preliminary information.

a) Some notions of control theory

b) Nonlocal exact controllability of 3D Navier-Stokes system

c) Few words on stabilisation by feedback control

(2) Local stabilisation of Navier-Stokes system

a) Setting of problem

b) Idea of proof

(3) Nonlocal stabilization of NPS connected to the 3D Helmholtz system

a) From Navier-Stokes to Helmholtz system

b) NPS: definition and properties

c) Nonlocal stabilization of NPS

(4) On nonlocal stabilization problem for one hydrodynamic type system

a) NPS corresponding to differentiated Burgers equation

b) Functional polar coordinates

c) Nonlocal stabilization of Burgers type equation

References:

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