

# Newton-multigrid FEM solver for the simulation of thixo-viscoplastic flow problems

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## ABSTRACT

This contribution is concerned with the application of Finite Element Method (FEM) and Newton-Multigrid solver to simulate thixo-viscoplastic flows.

The thixotropy phenomena are introduced to viscoplastic material by taking into consideration the internal material micro structure using a structure parameter. Firstly, the viscoplastic stress is modified to thixo-viscoplastic stress dependent on the structure parameter. Secondly, an evolution equation for the structure parameter is introduced to induce the time-dependent process of competition between the destruction (breakdown) and the construction (buildup) inherent in the material.

The thixo-viscoplastic stress is integrated, in quasi-Newtonian manner, into the generalized Navier-Stokes equations which constitute together with the evolution equation for the structure parameter the main core of full set of modeling equations. A fully coupled monolithic finite element approach has been applied to solve the material internal micro structure parameter, velocity, and pressure fields simultaneously. The non-linearity of the problem is treated with generalized Newton's method w.r.t. the Jacobian's singularities having a global convergence property. The linearized systems inside the outer Newton loops are solved using the geometrical multigrid methods with a Vanka-like smoother taking into account a stable FEM approximation pair for velocity and pressure.

Three configurations are systematically chosen to analyze the accuracy, robustness and efficiency of the Newton-Multigrid FEM solver and thixo-viscoplastic phenomena. Firstly, we used Lid-driven cavity benchmark to analyze the optimal setting, mesh refinement, and regularization. Indeed, we achieved a point-wise mesh convergence as well as a resolution barrier,  $(k, L)$  regularization and mesh refinement level, beyond which no further resolution's improvement is possible. Additionally, we showed the mesh refinement independency of the solver. Secondly, we considered the couette device configuration. As a model validation tool, we investigated the solid/liquid transition w.r.t. breakdown parameter. Indeed, some characteristics of thixotropic flows were captured, i.e. localization, shear banding, and discontinuous jump to infinity for viscosity. Thirdly, we used the most common industrial configuration, a 4:1 contraction, to analyze the impact of thixotropic phenomena. We showed that thixotropy inherent in the material induce more breakdown layers in the vicinity of the wall suggesting reduction of optimal pressure drop in this type of configurations.

## REFERENCES

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