Elliptic Problems in Lipschitz Domains

Chérif Amrouche[1], Mohand Moussaoui[2],

SUMMARY

We are interested here in questions related to the **maximal regularity** of solutions of **elliptic** problems with **Dirichlet** or **Neumann** boundary condition (see ([1]). For the last 30 years, many works have been concerned with questions when Ω is a **Lipschitz domain**.

We give here new proofs and some complements for the case of the **Laplacian** (see [3]), the **Bilaplacian** ([2] and [6]) and the operator $\mathrm{div}\,(A\nabla)$ (see ([5]), when **A** is a matrix or a function. And we extend this study to obtain other regularity results for domains having an adequate regularity. We give also new results for the **Dirichlet-to-Neumann** operator for Laplacian and Bilaplacian.

Using the duality method, we can then revisit the work of Lions-Magenes [4], concerning the so-called **very weak solutions**, when the data are less regular.

Keywords: Elliptic problems, Lipschitz domains, maximal regularity, Steklov Poincaré operator.

AMS Classification: 35C15, 35J25, 35J40

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[1]Laboratoire de Mathématiques et Leurs Applications, UMR CNRS 5142
Université de Pau et des Pays de l'Adour
email: `cherif.amrouche@univ-pau.fr`

[2]Lab. des EDP Non Linéaires et Histoire des Mathématiques
Ecole Normale Supérieure de Kouba, Alger
email: `mmohand47@gmail.comy`