Computation of Unsteady Flows With Moving Grids

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Unsteady Flows With Moving Boundaries, I

- Unsteady flows with moving boundaries can be analysed using various coordinate systems and base vectors.
- The simplest equations are obtained when the analysis is performed using a fixed coordinate system (laboratory frame) and Cartesian base vectors.
- The governing equations (conservation of mass, momentum, and scalar quantities, respectively) in integral form read:

Mass $$\frac{\partial}{\partial t}\int_V \rho \,\mathrm{d}V + \int_S \rho \,\mathbf{v}\cdot\mathbf{n}\,\mathrm{d}S = 0$$

Momentum $$\frac{\partial}{\partial t}\int_V \rho\mathbf{v}\,\mathrm{d}V + \int_S \rho\mathbf{v}\,\mathbf{v}\cdot\mathbf{n}\,\mathrm{d}S = \int_S (\mathbf{T} - p\mathbf{I})\cdot\mathbf{n}\,\mathrm{d}S + \int_V \rho\mathbf{b}\,\mathrm{d}V$$

Scalars $$\frac{\partial}{\partial t}\int_V \rho\phi\,\mathrm{d}V + \int_S \rho\phi\,\mathbf{v}\cdot\mathbf{n}\,\mathrm{d}S = \int_S \Gamma\nabla\phi\cdot\mathbf{n}\,\mathrm{d}S + \int_V \rho b_\phi\,\mathrm{d}V$$

Unsteady Flows With Moving Boundaries, II

- The first term on the left-hand side describes the rate of change in a *fixed control volume* and the second stands for the convective transport by fluid motion.
- The right-hand side contains diffusive fluxes and source terms.

$$\frac{\partial}{\partial t}\int_V \rho\phi \,\mathrm{d}V + \int_S \rho\phi\,\mathbf{v}\cdot\mathbf{n}\,\mathrm{d}S = \int_S \Gamma\nabla\phi\cdot\mathbf{n}\,\mathrm{d}S + \int_V \rho b_\phi\,\mathrm{d}V$$

- When studying flows around moving bodies, one can often reduce the complexity of the problem by using a moving (body-fixed) coordinate system.
- For example, flow around a moving body is unsteady when viewed from a fixed coordinate frame (i. e. for an observer who does not move), but it may be steady when viewed from a coordinate system attached to the body and moving with it.

Unsteady Flows With Moving Boundaries, III

- If the coordinate system itself moves, the momentum equation obtains on the left-hand side the following extra terms:

$$\int_V \rho \left[\mathbf{a}_0 + \left(\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} \times \mathbf{r} \right) + \left[\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \right] + (2\,\boldsymbol{\omega} \times \mathbf{v}) \right] \mathrm{d}V$$

- Here $\mathbf{a}_0$ is the acceleration of the coordinate system origin and $\boldsymbol{\omega}$ is the angular velocity vector describing the rotation of the coordinate system.

- All these extra terms vanish if the body moves linearly with a constant velocity; in that case equations are the same for both fixed and moving coordinate system.

- The difference lies in the meaning of the velocity vector $\mathbf{v}$: in the case of a moving coordinate system it represents the velocity relative to the body, which is equal to zero at body surface (no-slip boundary condition).

Unsteady Flows With Moving Boundaries, IV

- One can in such a situation easily convert relative velocity into absolute velocity and vice versa by adding or subtracting the velocity of the coordinate system:

$$\mathbf{v}_{\text{abs}} = \mathbf{v}_{\text{rel}} + \mathbf{v}_{\text{cs}}$$

- The analysis in relative terms is often used in experiments as well as in numerical studies...
- In a wind tunnel, the car (or another vehicle) model is stationary and the air (and – if one wants to be fully correct – the floor) is moving...
- Commercial CFD-codes offer the option of using a „moving frame of reference (MRF)“...
- … which is often used to simplify the analysis and avoid grid motion and transient analysis.

Unsteady Flows With Moving Boundaries, V

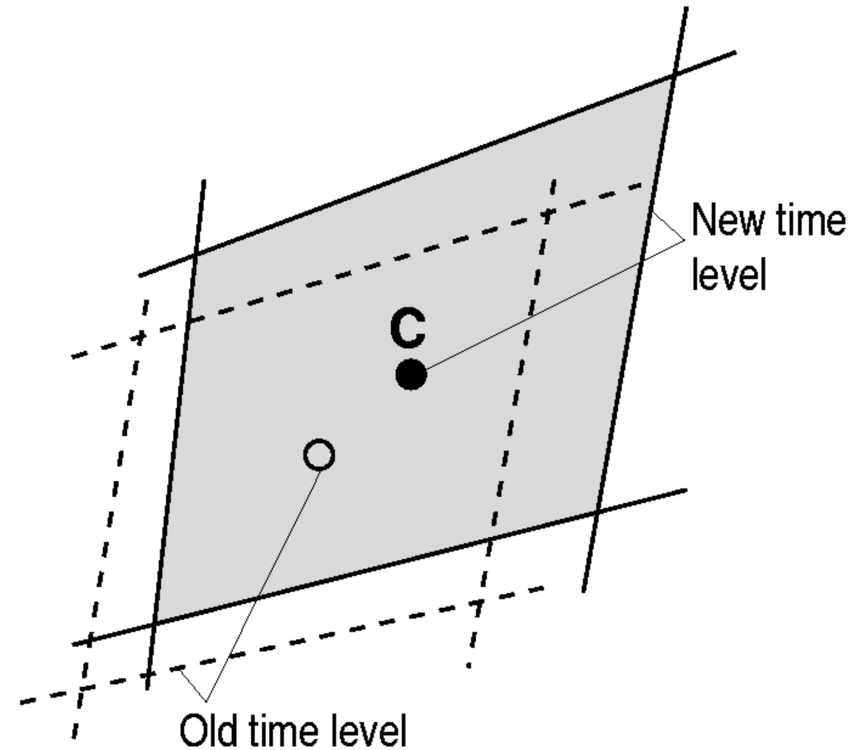
- The MRF-concept can be applied region-wise: in one part of the grid around moving body, moving reference frame is used...
- … while in the rest, fixed reference frame is used.
- Correct solution is obtained if at the interface between the two reference frames the solution is steady in both reference frames.
- This approach can deliver large errors if applied where it is not appropriate (e.g. when interaction between moving and fixed parts is important)...
- Example:
  - Flow around a propeller in an “open-water test” can be analysed as steady in a moving reference frame;
  - Flow around a propeller behind ship hull requires actual motion of propeller relative to hull to properly account for interaction...

Modelling of Small Boundary Motion

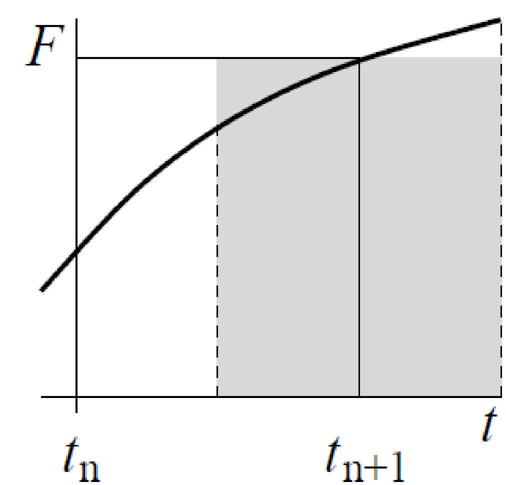
- If the boundary displacement is small relative to grid size, one can simulate it by applying mass sources or sinks...
- This is appropriate for high-frequency, small amplitude wall motion (e.g. piezo-electric actuators, wall vibration etc.).
- The wall is then held fixed, but mass source or sink is applied corresponding to the amount of displaced/sucked fluid by wall motion.
- The source term is computed as if the fluid was flowing into or out of control volume with the wall velocity (the wall-normal component).
- Wall boundary condition has to be applied for the tangential direction (i.e. one cannot treat the boundary as “inlet boundary” - shear stress needs to be accounted for).

Moving Control Volumes, I

- If the body motion is irregular, the flow is unsteady from any viewpoint and in this case one cannot simplify the problem...
- There are several options for numerical simulation of such flows, both with respect to equations and to grid used...
- The grid has to move, but one can still use equations for a fixed control volume if a fully-implicit time-integration method is used (i.e. fluxes and source terms evaluated only at the new time level).
- For example, with a 2nd-order quadratic backward scheme:

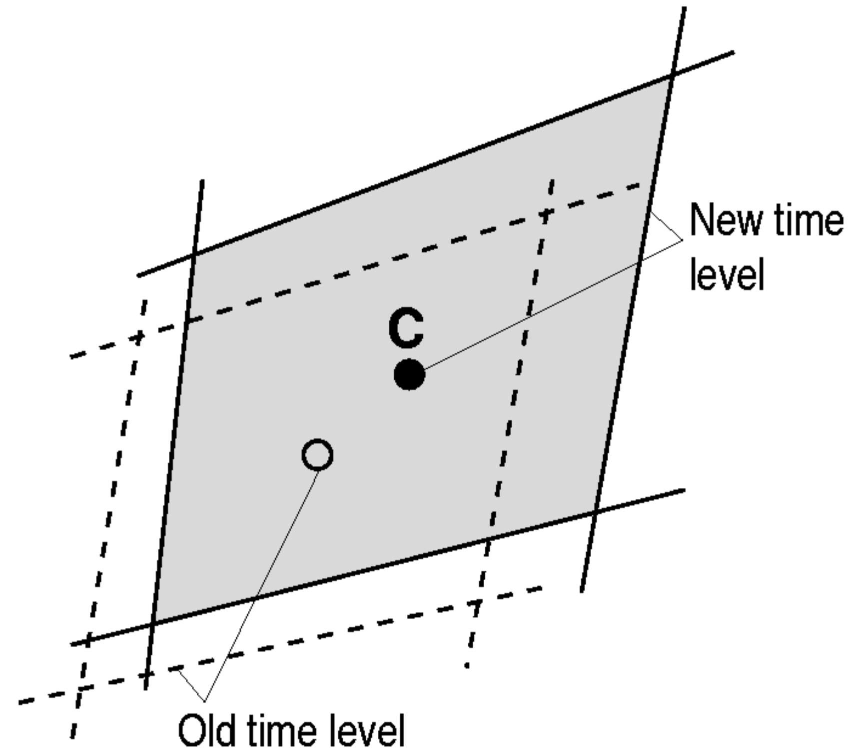


$$\frac{\partial}{\partial t}\int_V \rho\phi\,\mathrm{d}V \approx \frac{\left[3(\rho\phi)^n_{C^n} - 4(\rho\phi)^{n-1}_{C^n} + (\rho\phi)^{n-2}_{C^n}\right]\,\Delta V}{2\,\Delta t}$$



Moving Control Volumes, II

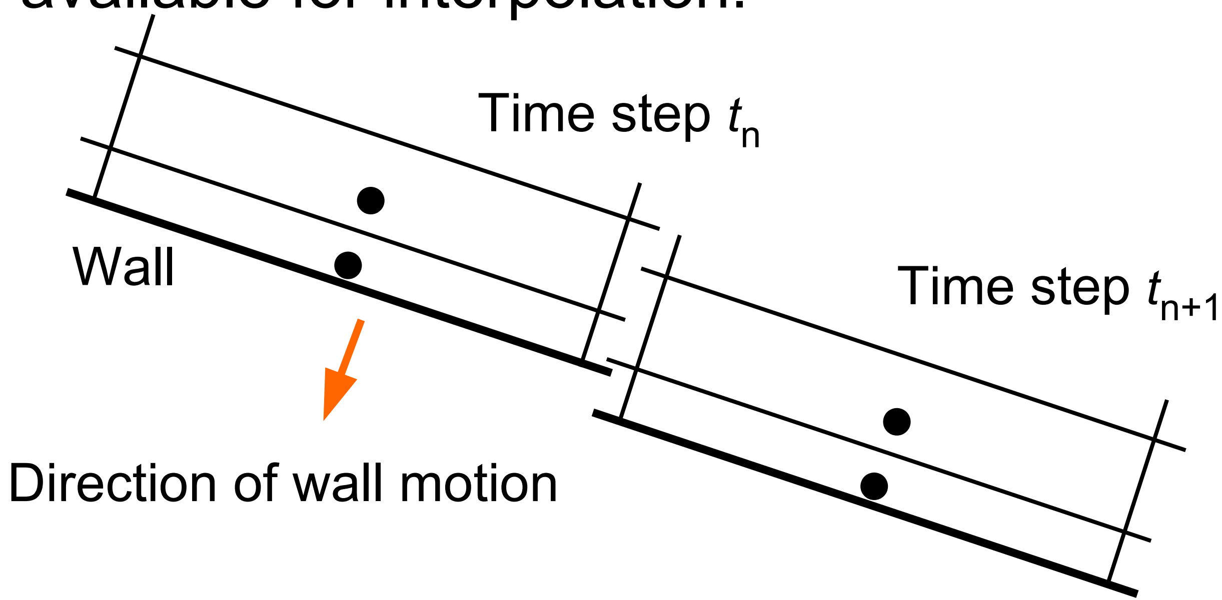
- The grid moves, but when a fully-implicit time-integration scheme is used, the motion affects only the rate-of-change term...
- Convective fluxes due to wall motion feature in equations (fluid is displaced by wall – the analysis is performed in a fixed control volume at the new location)...
- Interpolation of old solutions to the new cell-center location introduces additional discretization errors...
- One possibility is linear interpolation, but higher order methods are better...



$$\phi_{C^n}^{n-1} \approx \phi_{C^{n-1}}^{n-1} + \left(\nabla\phi\right)_{C^{n-1}}^{n-1} \cdot \left(\mathbf{r}_{C^n} - \mathbf{r}_{C^{n-1}}\right)$$

Moving Control Volumes, III

- The interpolation errors can be reduced by using higher-order approximations…
- The problem is the motion of walls in the direction normal to boundary…
- … because here cells are usually very thin (prism layer to account for high gradients)…
- … and it can happen that cell centers at the new time step fall outside solution domain from previous time step – no old solution available for interpolation.



This approach has been implemented and tested by Hidajet Hadžić at TUHH; for details see his PhD thesis:

http://doku.b.tu-harburg.de/volltexte/2006/296/index.html

Moving Control Volumes, IV

- Another option is to use equations formulated for a moving control volume:

Mass: $$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho \,\mathrm{d}V + \int_S \rho(\mathbf{v}-\mathbf{v}_\mathrm{b})\cdot\mathbf{n}\,\mathrm{d}S = 0$$

Momentum: $$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho\mathbf{v}\,\mathrm{d}V + \int_S \rho\mathbf{v}(\mathbf{v}-\mathbf{v}_\mathrm{b})\cdot\mathbf{n}\,\mathrm{d}S = \int_S (\mathbf{T}-p\mathbf{I})\cdot\mathbf{n}\,\mathrm{d}S + \int_V \rho\mathbf{b}\,\mathrm{d}V$$

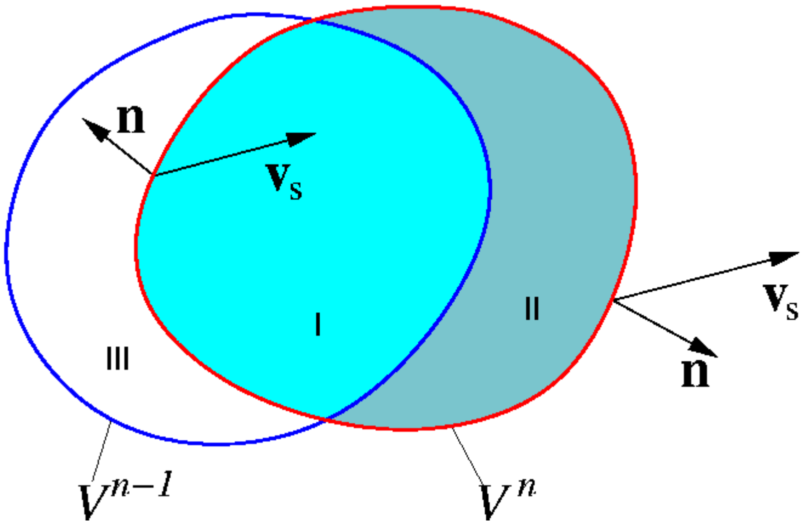
Scalars: $$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho\phi\,\mathrm{d}V + \int_S \rho\phi(\mathbf{v}-\mathbf{v}_\mathrm{b})\cdot\mathbf{n}\,\mathrm{d}S = \int_S \Gamma\nabla\phi\cdot\mathbf{n}\,\mathrm{d}S + \int_V \rho b_\phi\,\mathrm{d}V$$

The meaning of the time derivative is now different: it describes the change of transported quantity in time related to different locations, so it is no longer a local (partial) derivative...

This is reflected by the use of relative velocity in the convective term...

Moving Control Volumes, V

- In the event that the control volume surface moves with exactly the same velocity as the fluid, then no fluid would ever leave the control volume and all convective fluxes would vanish...
- In such a case the *control volume* becomes *control mass* (Lagrangean analysis – the time derivative becomes the substantial derivative)...
- … and represent the rate of change along a trajectory of a fluid element...



Grid Motion, I

- In the case of moving grids, the space-conservation law (SCL) has also to be satisfied:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}V - \int_S \mathbf{v}_\mathrm{b}\cdot\mathbf{n}\,\mathrm{d}S = 0$$

- In the case of constant density, the mass-conservation equation becomes:

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}V - \int_S \mathbf{v}_\mathrm{b}\cdot\mathbf{n}\,\mathrm{d}S} + \int_S \mathbf{v}\cdot\mathbf{n}\,\mathrm{d}S = 0$$

- The framed part is SCL and it must be zero to ensure that the velocity field is divergence-free...

$$\int_S \mathbf{v}\cdot\mathbf{n}\,\mathrm{d}S = 0 \quad \text{or} \quad \nabla\cdot\mathbf{v} = 0$$

Grid Motion, II

- If SCL is not satisfied by the discretized equations, artificial mass sources or sinks will result...
- Note that grid motion is usually prescribed by imposed body motion, so grid velocity featuring in conservation equations is not an additional variable...
- Space conservation law is therefore not solved as a transport equation, but fluxes and volumes need to be computed so that it is satisfied…

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \mathrm{d}V - \int_S \mathbf{v}_\mathrm{b}\cdot\mathbf{n}\,\mathrm{d}S = 0$$

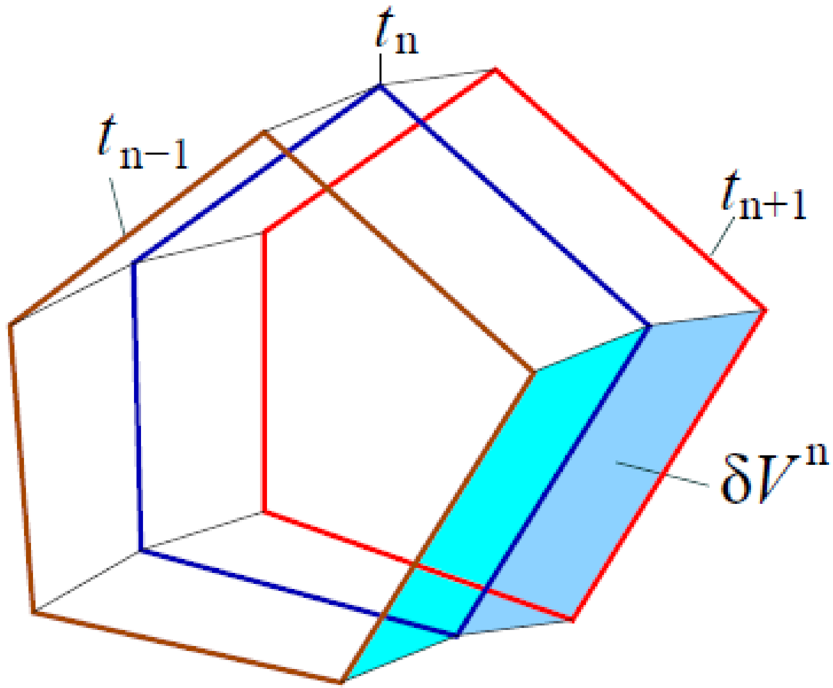
Grid Motion, III

- Discretized form of SCL, e.g. with the implicit three-time-levels scheme:

$$\frac{3\,\Delta V^{n+1} - 4\,\Delta V^{n} + \Delta V^{n-1}}{2\,\Delta t} = \sum_k \left[(\mathbf{v}_\mathrm{b} \cdot \mathbf{n})_k S_k \right]^{n+1}$$

The volume change from one time step to the other can be expressed through volumes swept by cell faces:

$$\Delta V^{n+1} - \Delta V^{n} = \sum_k \delta V_k^n$$



Grid Motion, IV

- The volume fluxes due to grid motion can be expressed through swept volumes as:

$$\dot{V}_k^{n+1} = \left(\int_{S_k} \mathbf{v}_\mathrm{b} \cdot \mathbf{n} \,\mathrm{d}S\right)^{n+1} \approx [(\mathbf{v}_\mathrm{b} \cdot \mathbf{n})_k S_k]^{n+1} \approx \frac{3\,\delta V_k^n - \delta V_k^{n-1}}{2\,\Delta t}$$

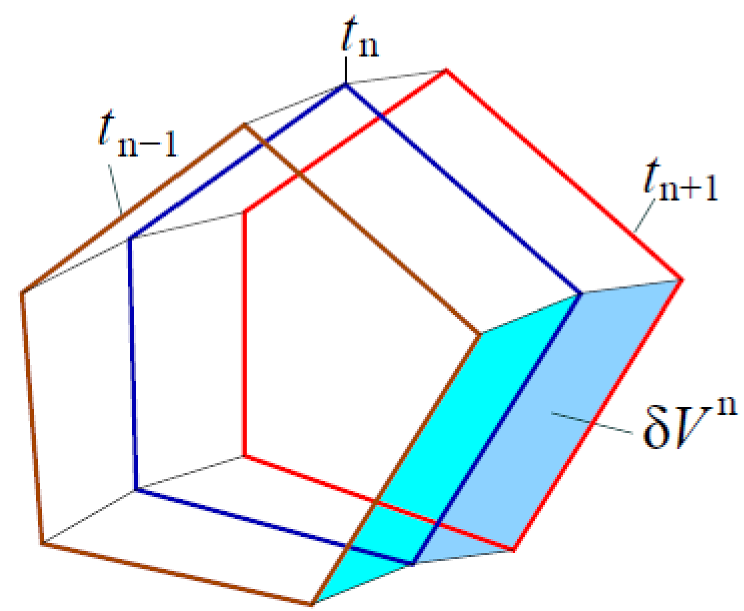
- This ensures that the SCL is satisfied automatically and the grid velocity need not be explicitly computed...
- Mass flux through CV-face can be approximated as:

$$\dot{m}_k^{m*} = \int_{S_k} \rho \mathbf{v} \cdot \mathbf{n} \,\mathrm{d}S - \int_{S_k} \rho \mathbf{v}_\mathrm{b} \cdot \mathbf{n} \,\mathrm{d}S \approx$$

$$(\rho v_n^{m*} S)_k - \rho_k \dot{V}_k^{m-1}$$

Contribution by fluid motion

Contribution by grid motion

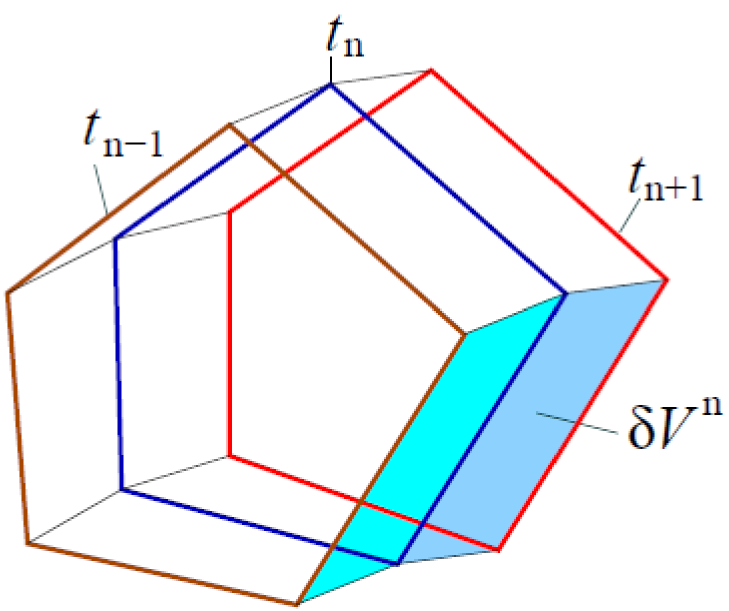


Grid Motion, V

- The discretized mass conservation equation:

$$\frac{3\left(\rho_{\mathrm{P}}\Delta V\right)^{m-1}-4\left(\rho_{\mathrm{P}}\Delta V\right)^{n}+\left(\rho_{\mathrm{P}}\Delta V\right)^{n-1}}{2\,\Delta t}+\sum_{k}\dot{m}_{k}^{m*}+\sum_{k}\dot{m}_{k}'=0$$

- If grid motion is part of the solution (e.g. flying or floating bodies), both density and volume representing new solution need to be updated…
- The computation of volume swept by faces during one time step and the computation of cell volume must be consistent…



Grid Motion, VI

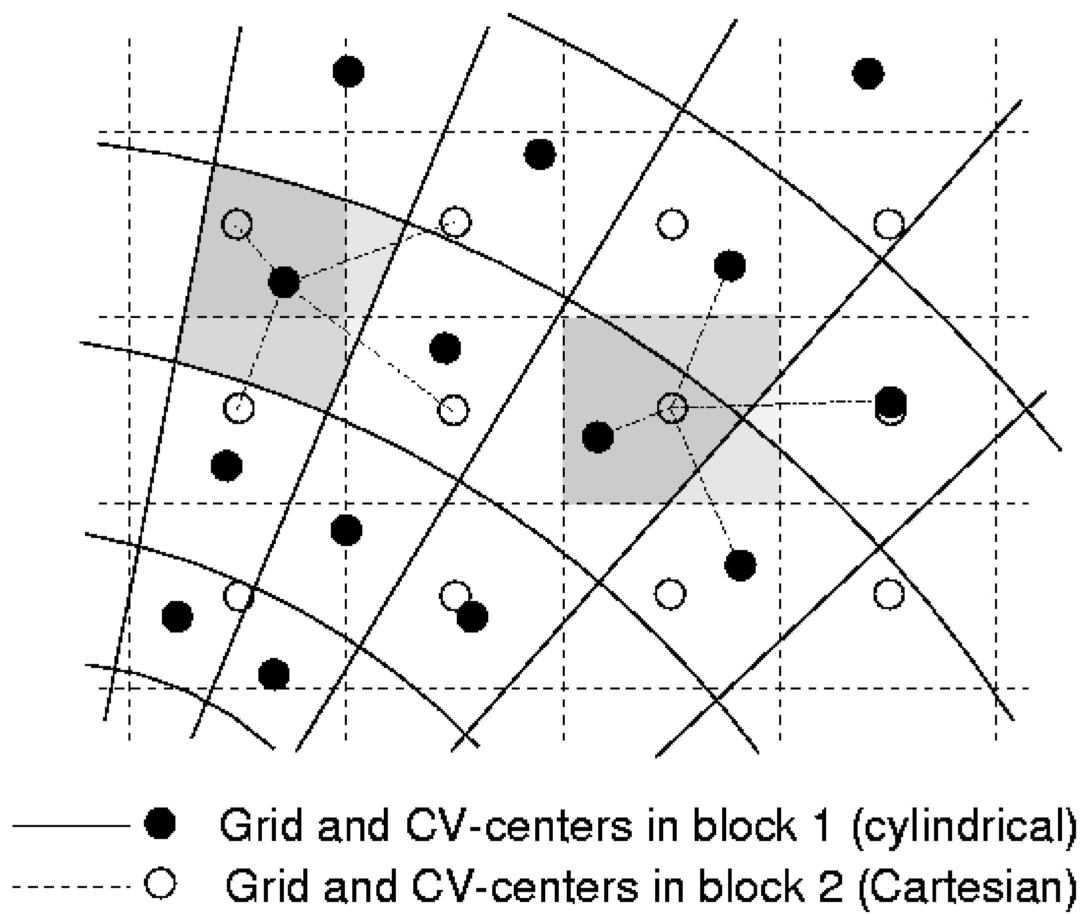
- The convective fluxes are zero at walls when the analysis is performed using moving control volumes...
- Fluid displacement by moving walls is taken into account through volume change...
- Large time steps can be taken, but temporal discretization errors will be large if cell faces move by one cell width or more...
- A good test of moving grid implementation: take a closed solution domain with no inlets/outlets, set fluid velocity to zero as initial condition, and just move the interior grid in each time step...
- If SCL is properly implemented, fluid will remain at rest – otherwise artificial mass sources or sinks will result and the fluid will start moving...

Grid Motion, VII

- The control of mesh motion is sometimes not trivial...
- One possibility: introduce a pseudo-solid in the flow domain, specify displacements of boundary nodes (or a slip condition) and compute displacements of inner grid nodes by solving equations governing solid body deformation...
- The problem: if the boundary moves too much, the original grid may deform too much and be no longer acceptable for computation...
- The solution: when the grid quality becomes poor, generate a new mesh and interpolate the solution to this mesh, then restart the simulation...
- Additional interpolation errors are introduced, even if interpolation is made conservative (but the errors are proportional to mesh spacing and are thus consistent with other discretization errors).

Grid Motion, VIII

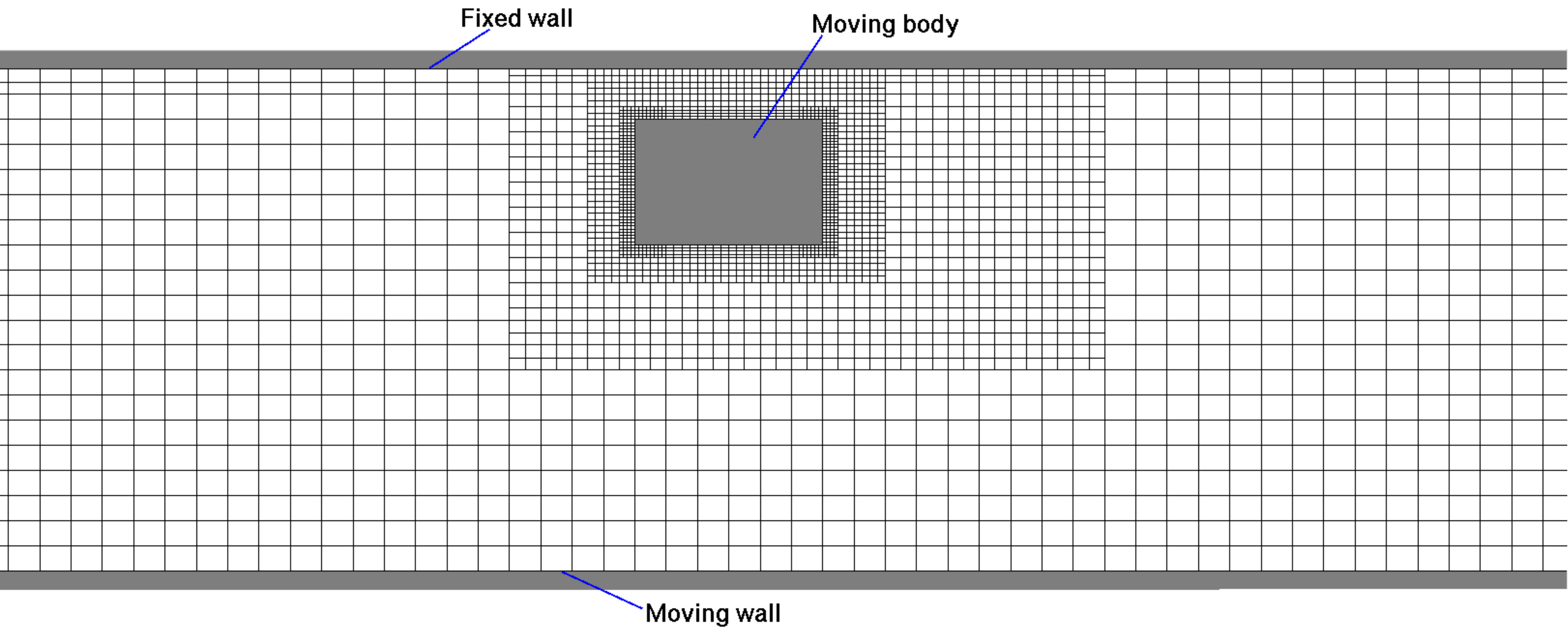
- In many applications, one can work with a combination of fixed and moving grids, with sliding interfaces between them...
- Sliding interface requires either hanging nodes or treating cells along interfaces as polyhedra...



Grid Motion, IX

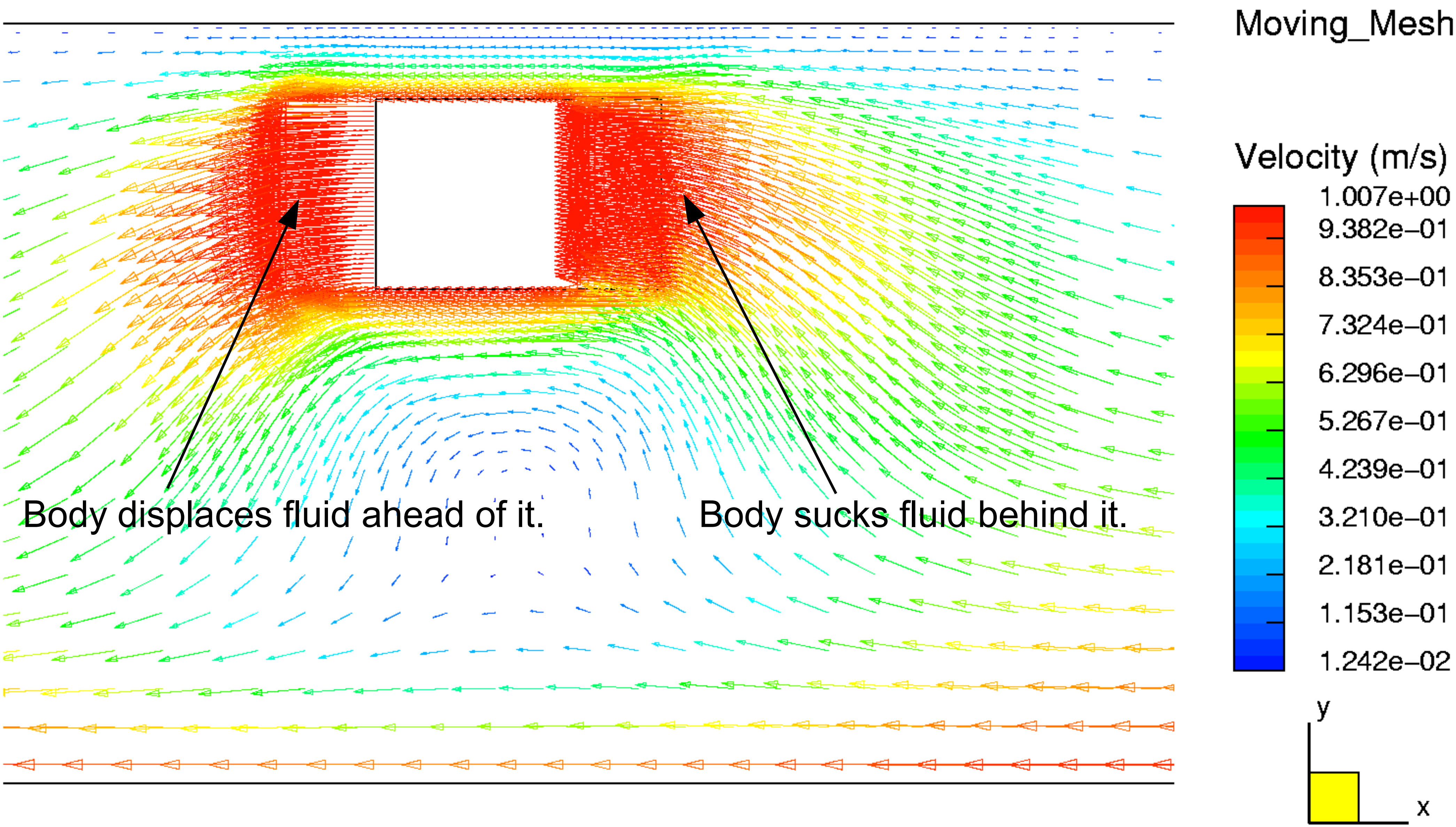
- Another possibility: the use of overlapping grids...
- Problems: coupling of the various grids, data management (grids may go out of physical flow domain or into solid regions...)
- Solution: efficient search algorithms and book-keeping, appropriate interpolation, strong coupling of grid blocks...
- Advantages: bodies can move arbitrarily relative to each other...
- Application areas: flying and floating bodies, rotating machinery...

Example of Flow Around a Moving Body, I



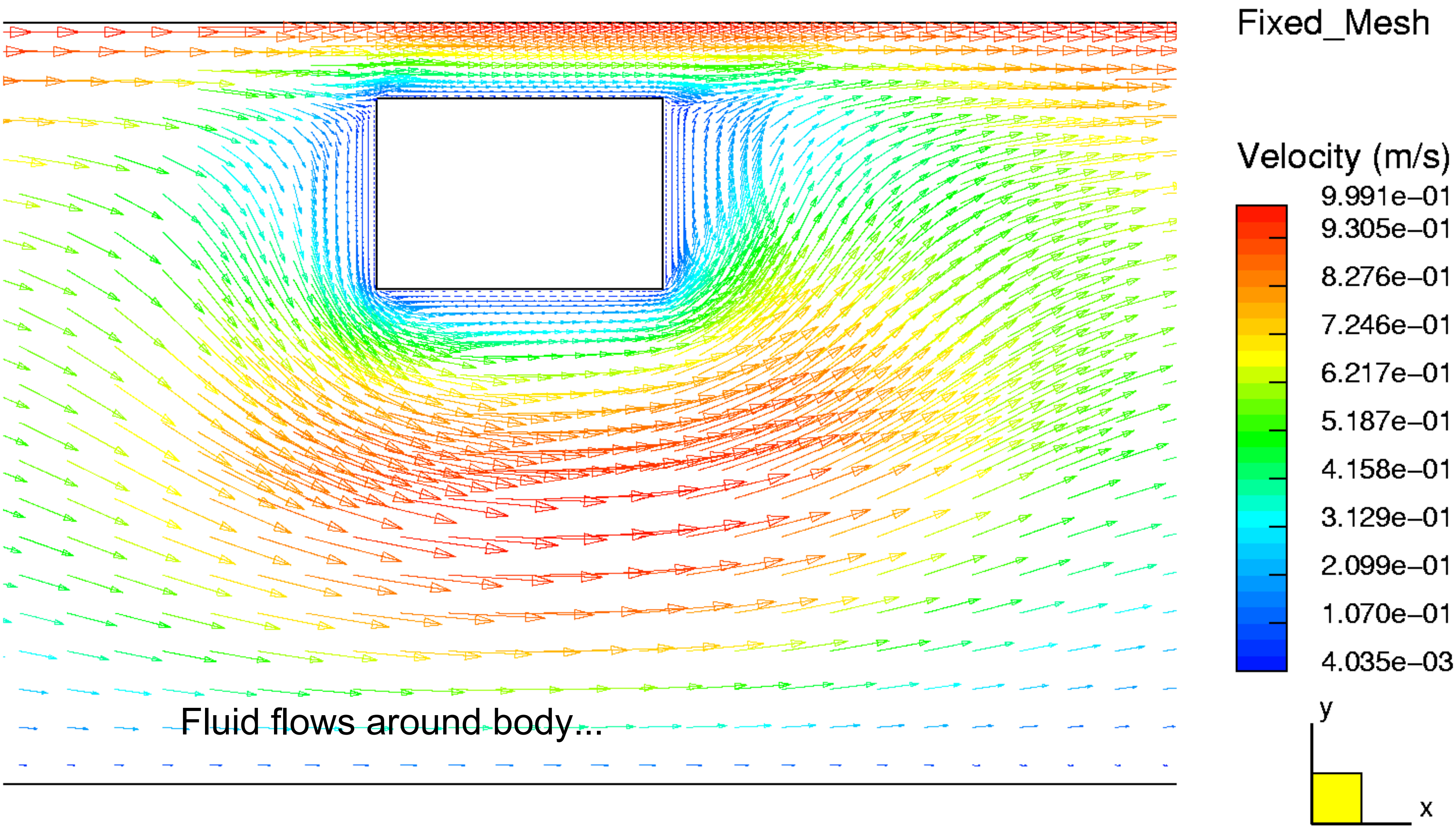
- Test case for flow around a body moving in a channel with constant velocity (from right to left): the mesh is locally refined around the body. The flow is laminar (Reynolds number of the order of 1). The solution domain is larger than the figure shows.

Example of Flow Around a Moving Body, II



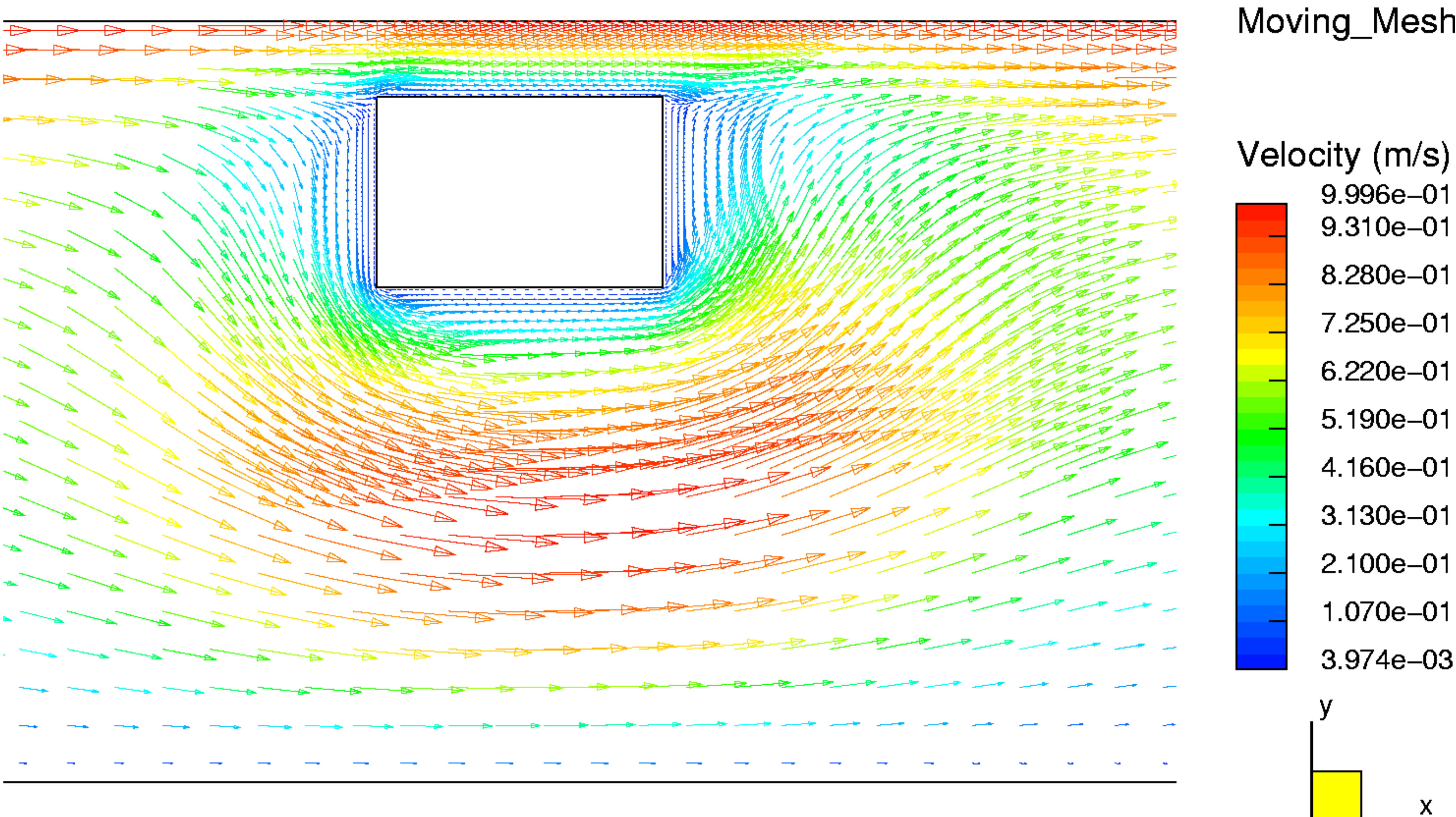
- Field of absolute velocity in body vicinity, computed using a moving mesh.

Example of Flow Around a Moving Body, III



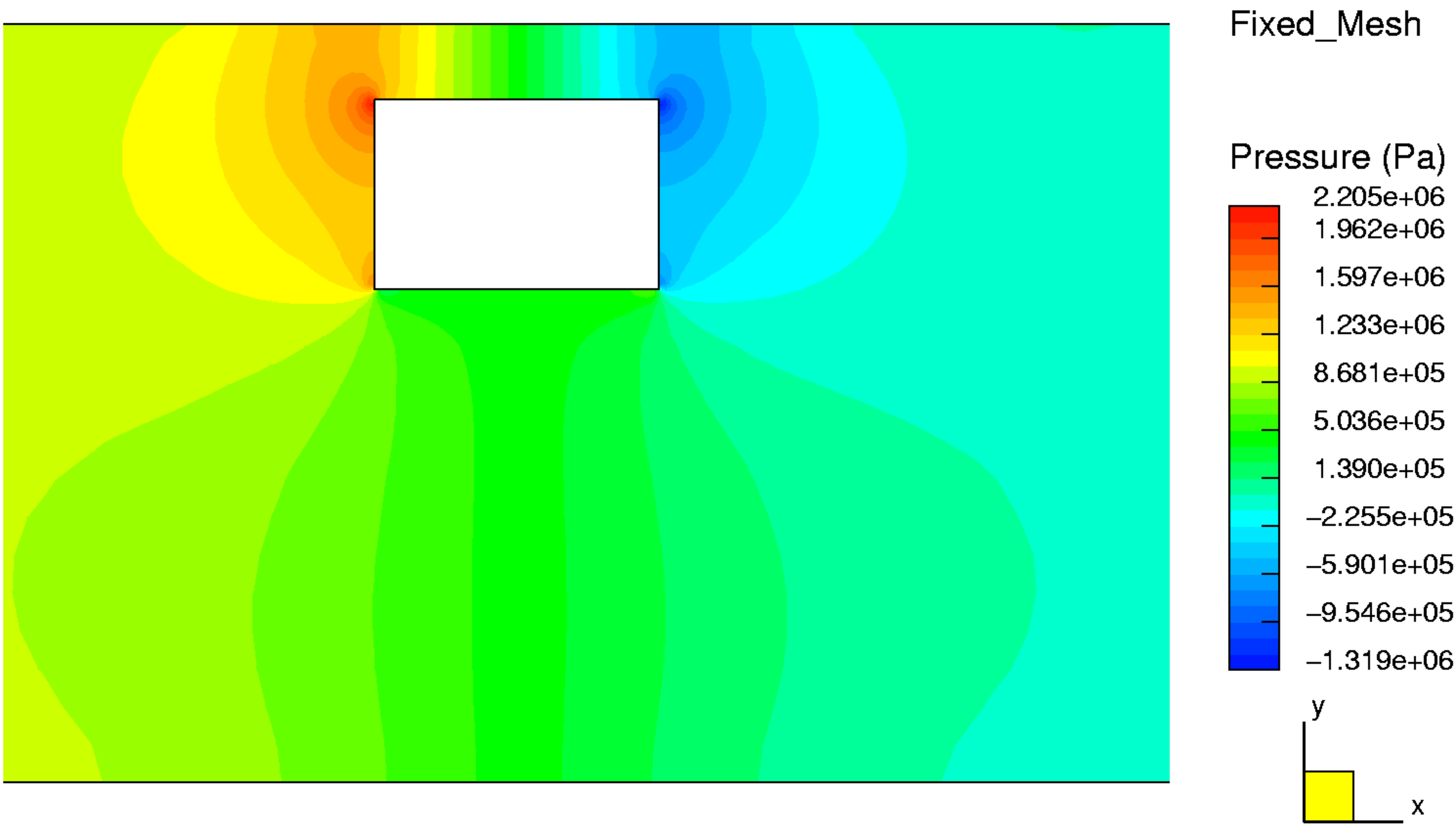
- Field of relative velocity in body vicinity, computed using a fixed mesh and a coordinate system moving with the body.

Example of Flow Around a Moving Body, IV



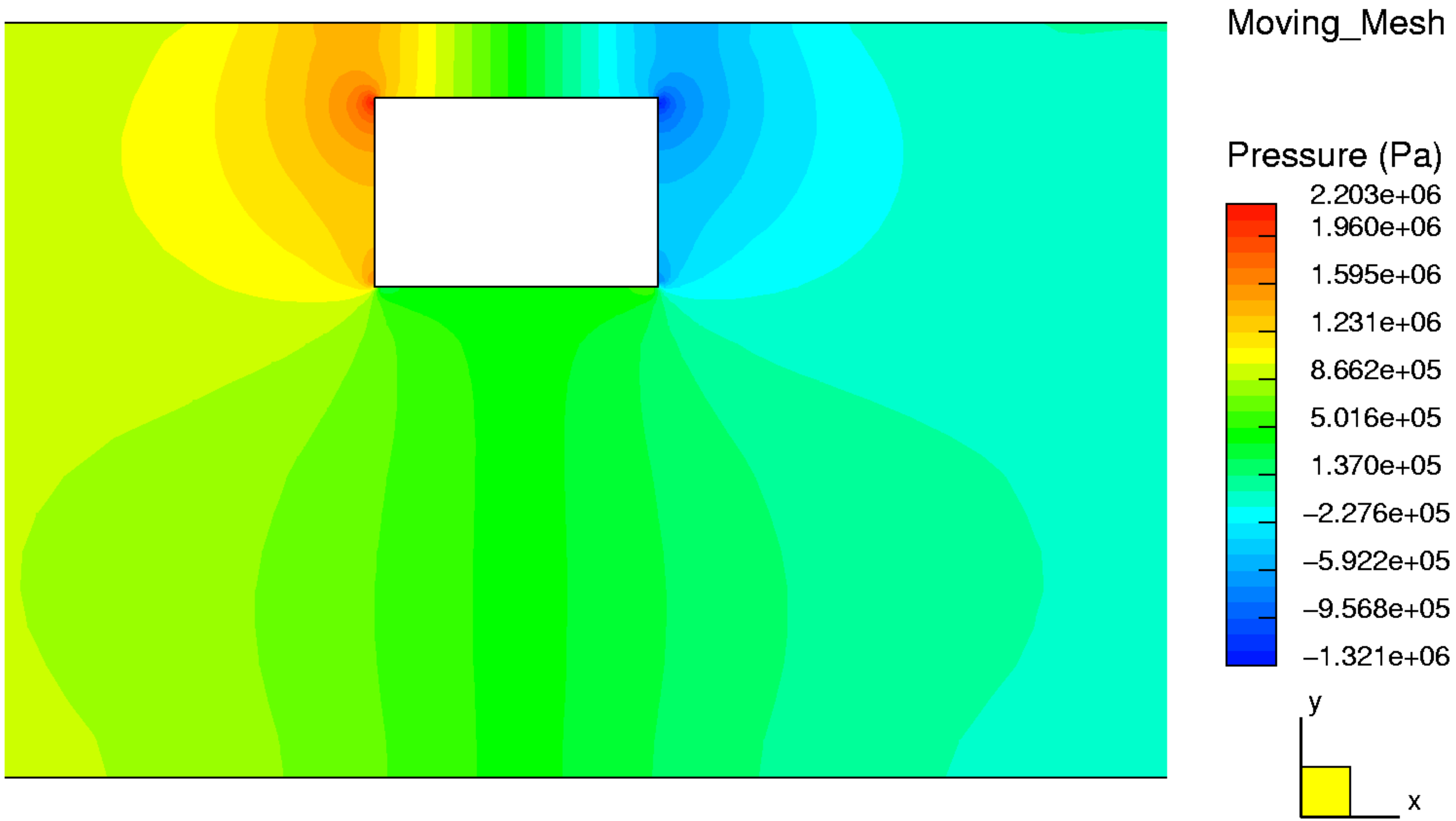
- Field of relative velocity in body vicinity, obtained by subtracting body velocity from the absolute velocity field computed using a moving mesh: almost identical to the result obtained in the moving reference frame...

Example of Flow Around a Moving Body, V



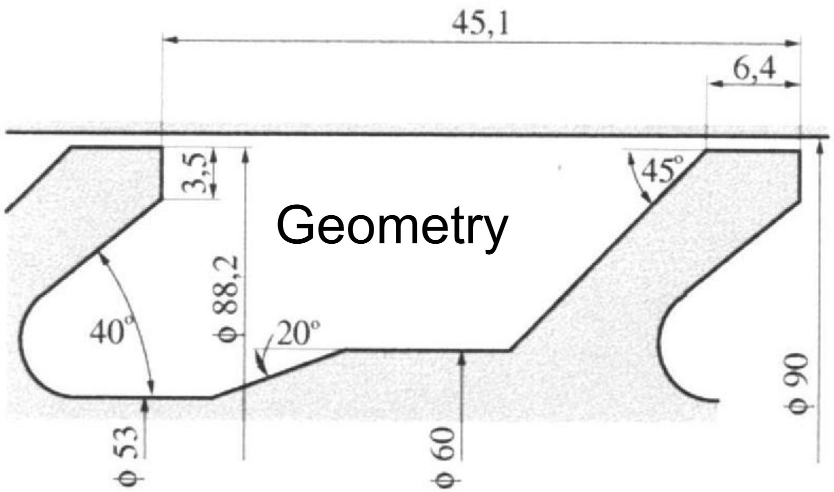
- Pressure field in body vicinity, obtained from a steady analysis in a body-fixed coordinate system.

Example of Flow Around a Moving Body, VI

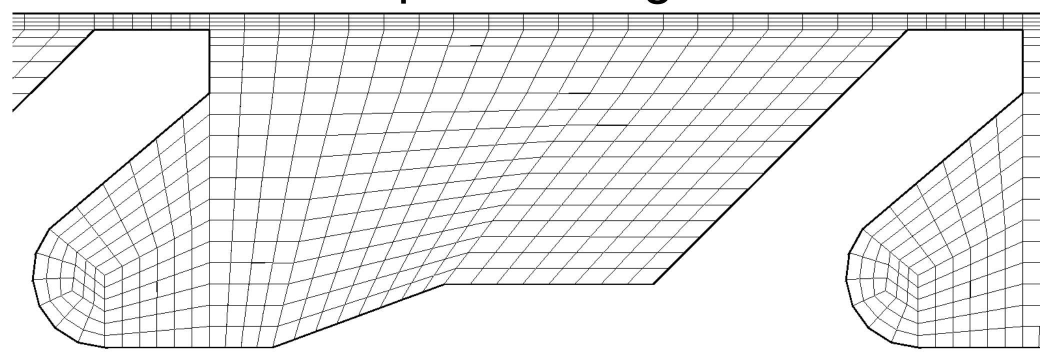


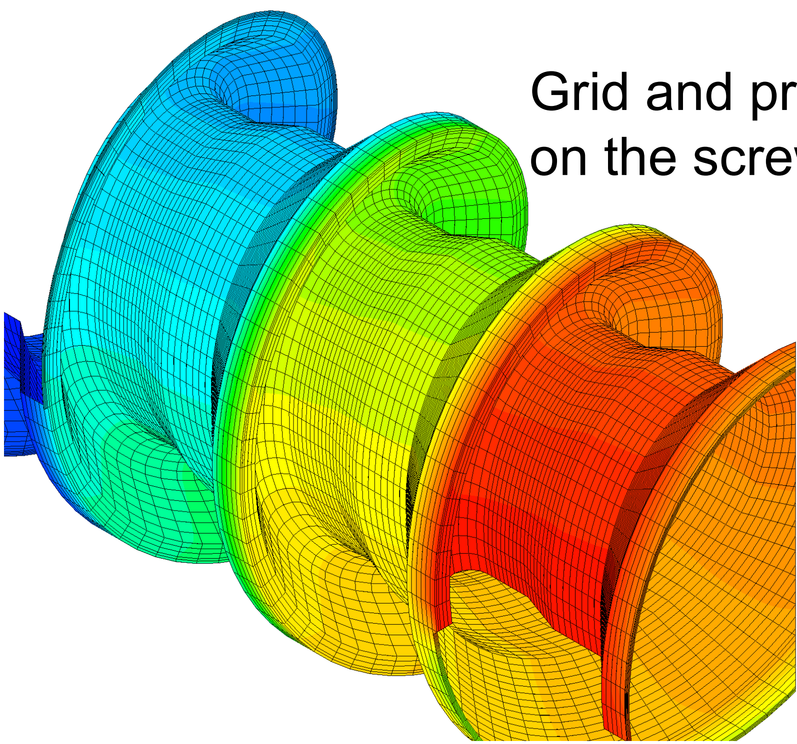
- Pressure field in body vicinity, computed in an unsteady analysis using a moving mesh – almost identical to the solution obtained using MRF...

Example, Single-Screw Extruder, I

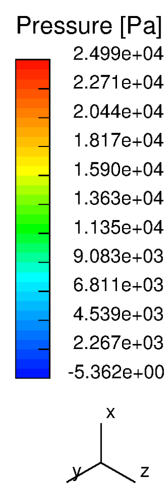


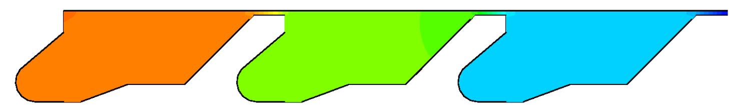
Computational grid



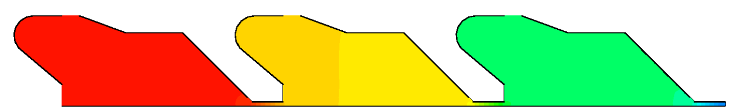


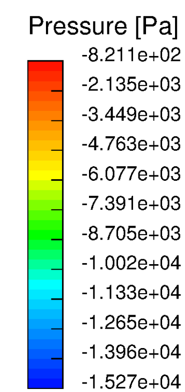
Grid and pressure on the screw





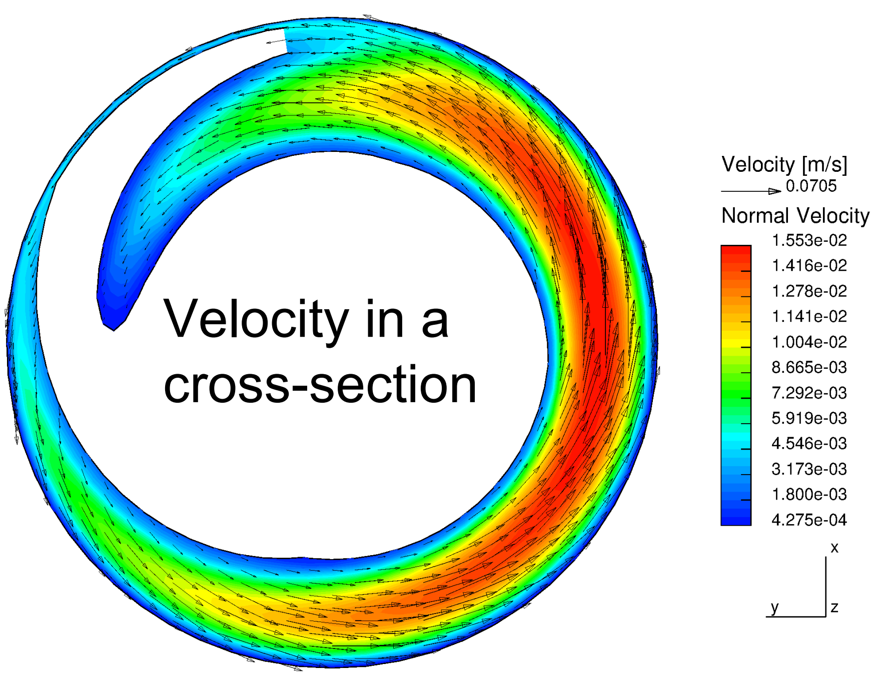
Pressure in an axial cross-section



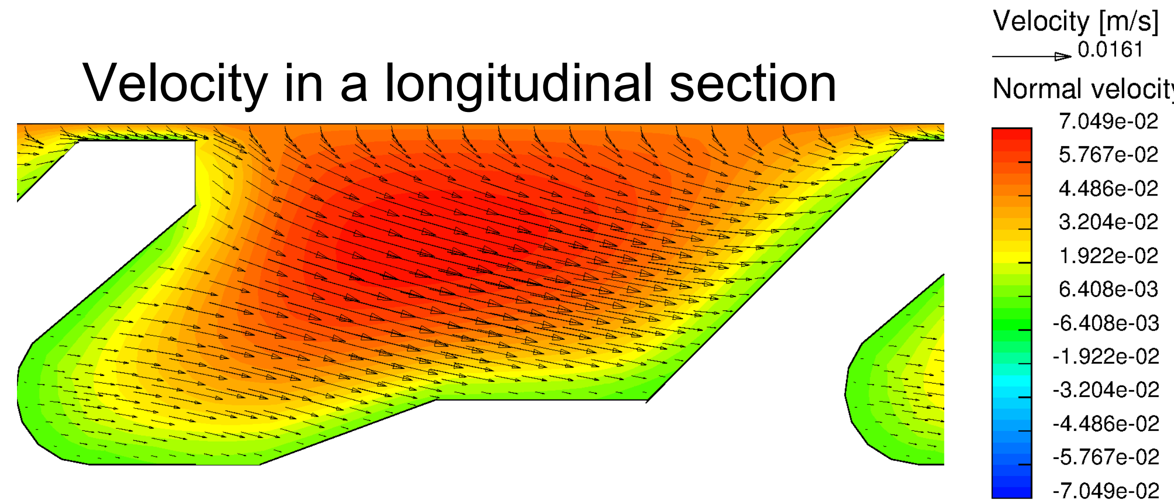


Fully developed flow assumed – periodic boundary conditions at inlet and outlet, computation in a rotating frame of reference…

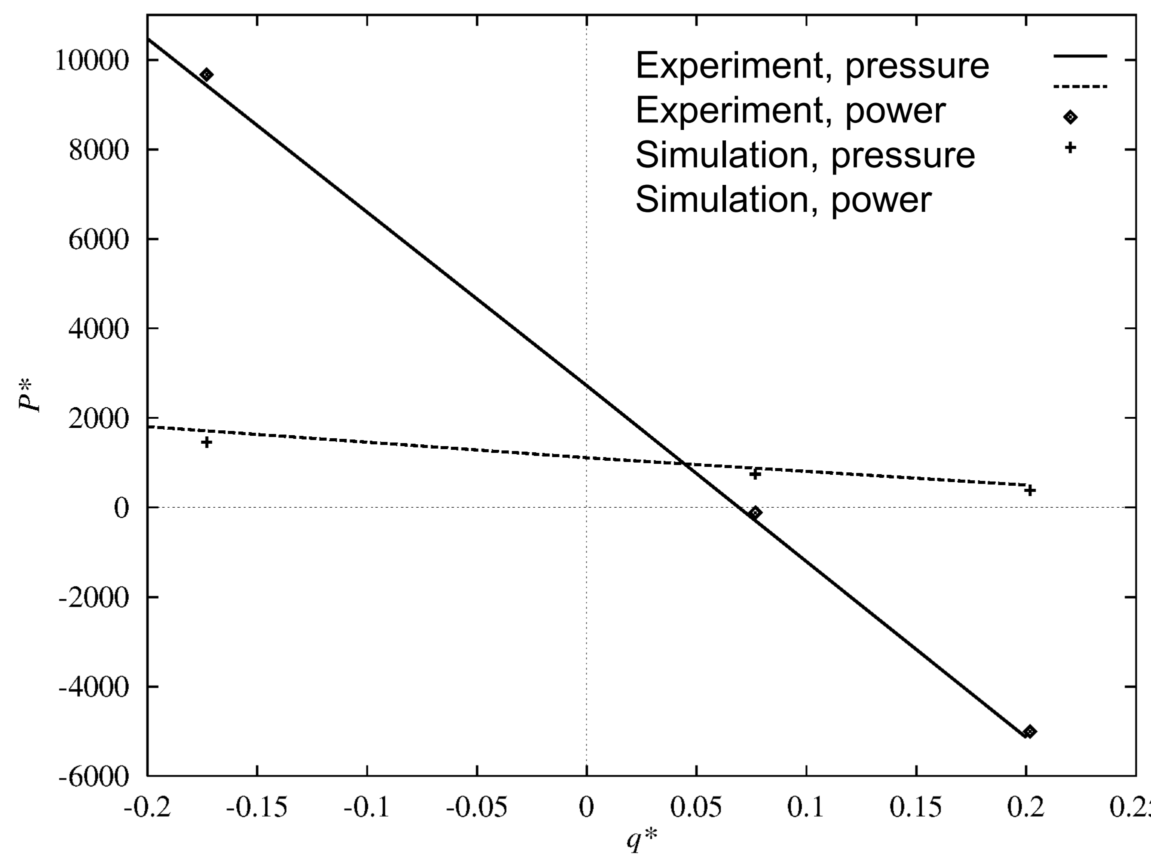
Example, Single-Screw Extruder II



Velocity in a longitudinal section

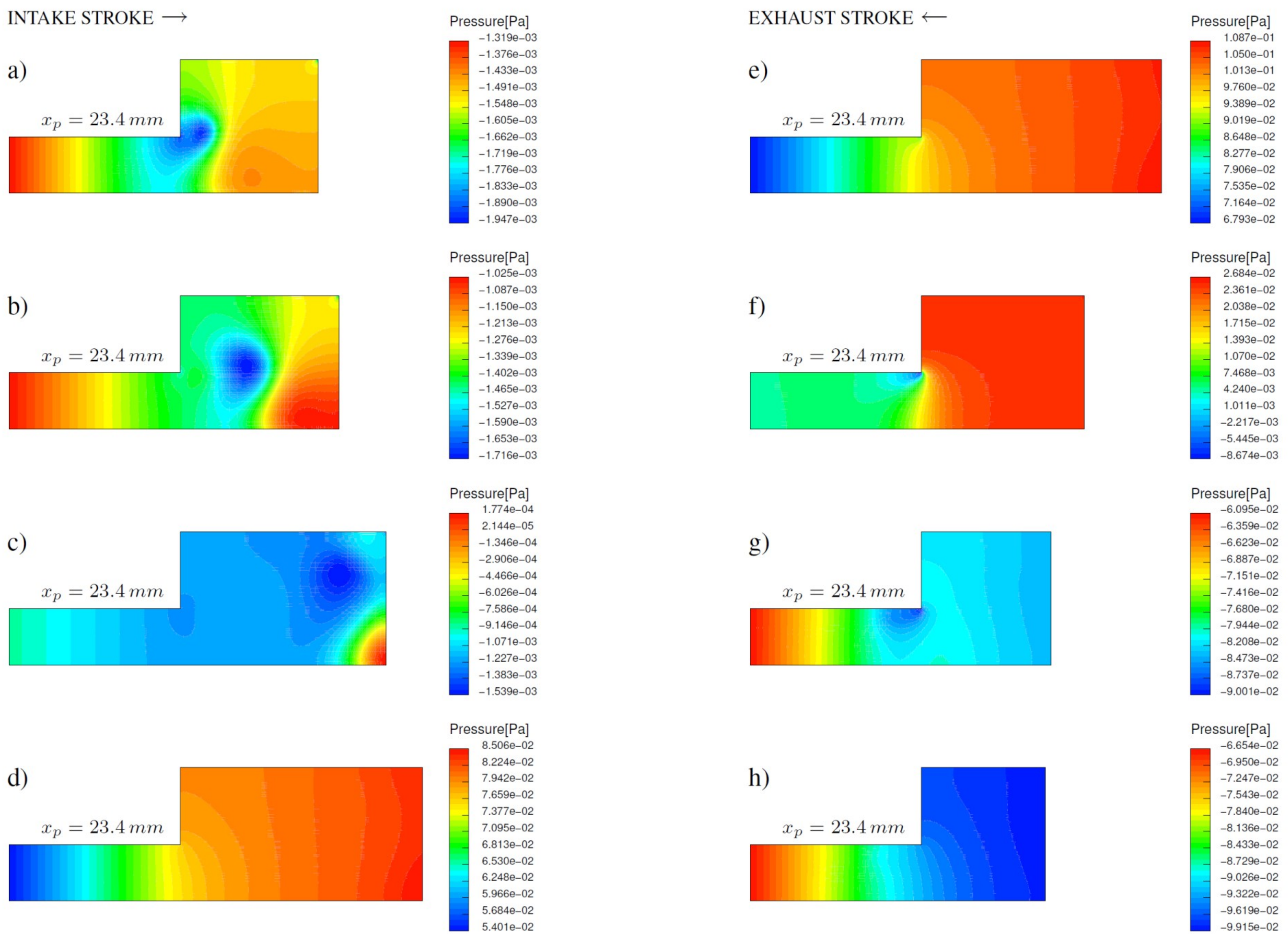


Comparison of predicted pressure drop and power input with experimental data for given flow rates – a very good agreement is achieved (laminar flow due to high viscosity, no further modeling errors).



Example, Piston-Driven Flow, I

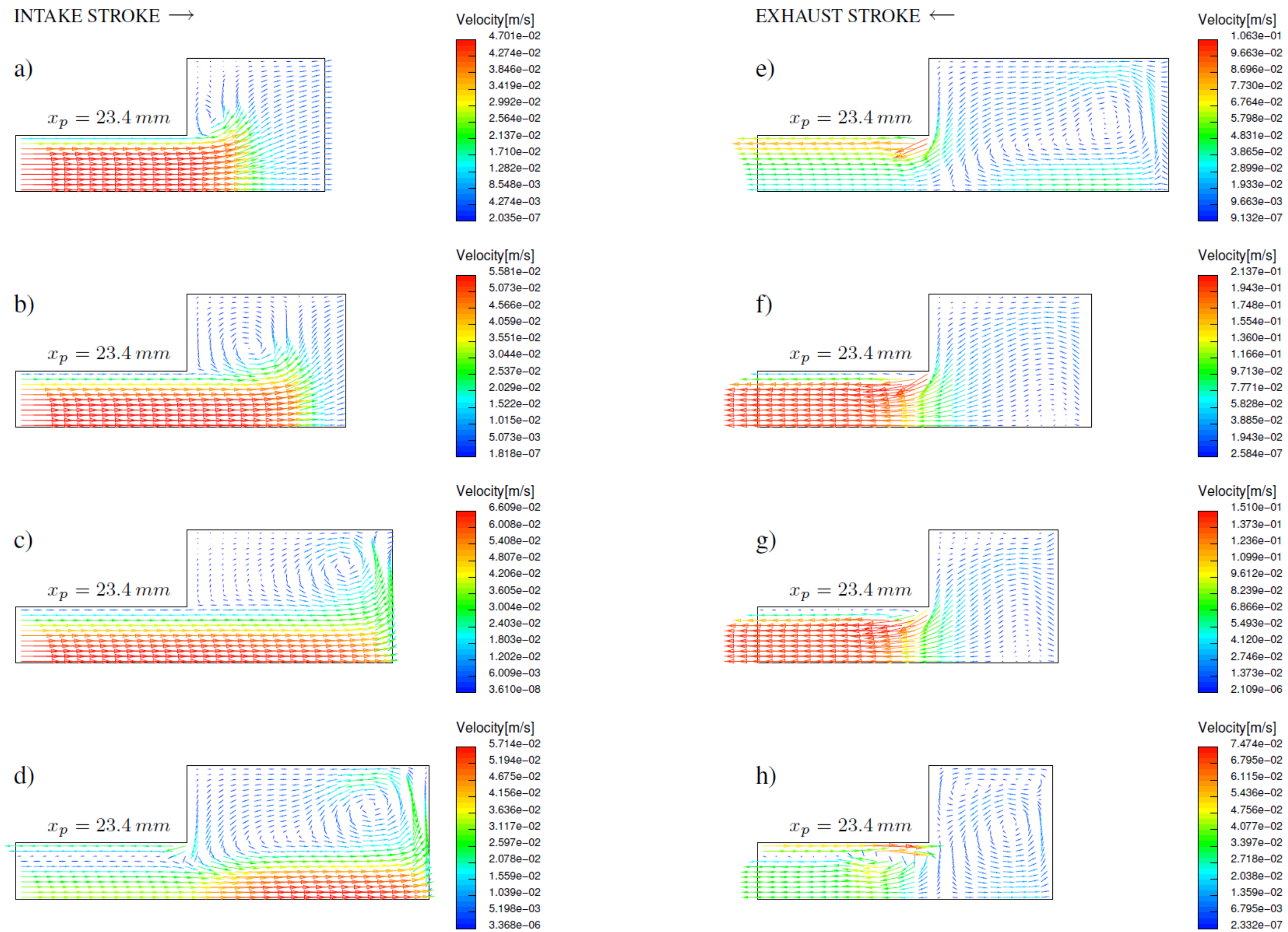
Simulation of intake and exhaust strokes in a simplified piston-cylinder assembly



From PhD Thesis by Hidajet Hadžić, TUHH, 2005

Example, Piston-Driven Flow, II

Simulation of intake and exhaust strokes in a simplified piston-cylinder assembly



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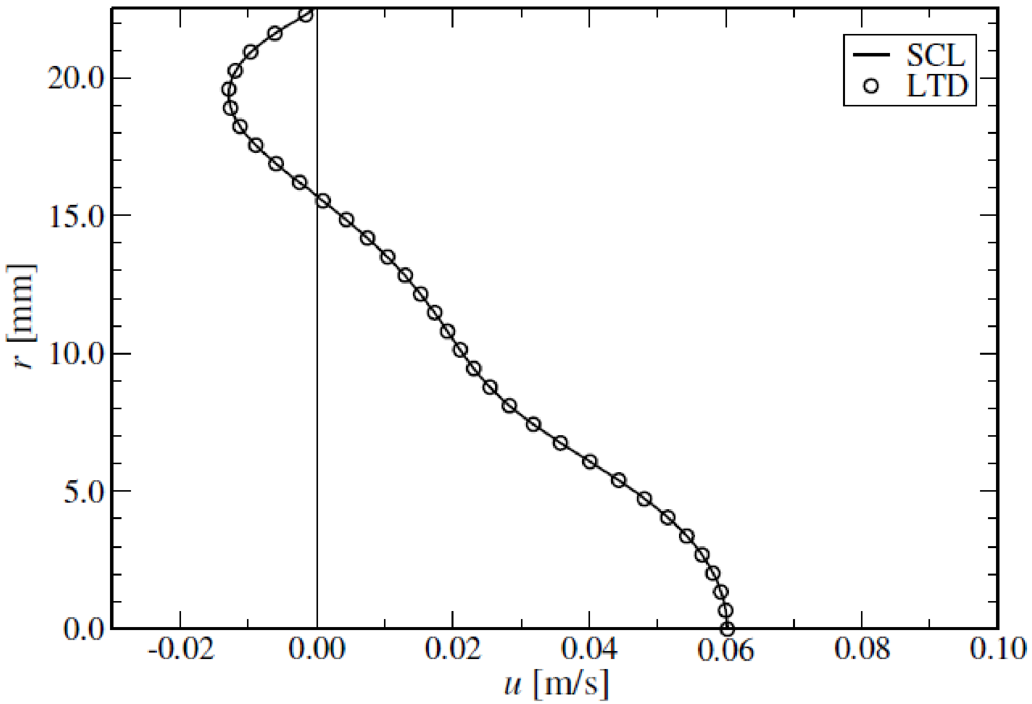
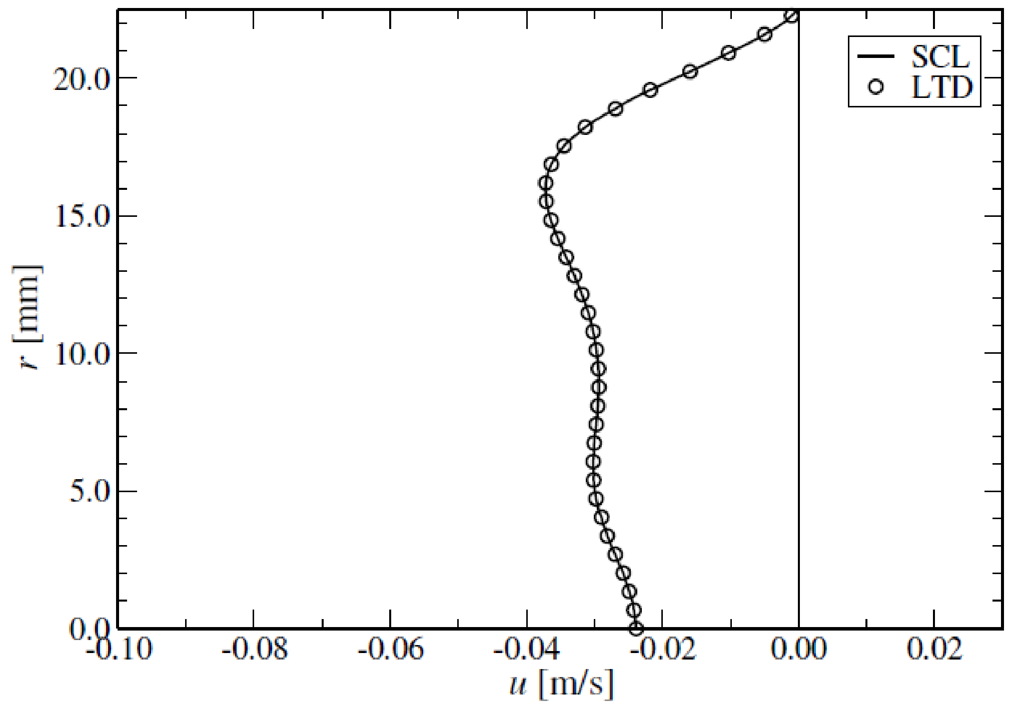
Example, Piston-Driven Flow, III

Intake stroke, x_p = 34.847 mm



Exhaust stroke, x_p = 26.876 mm



SCL: The grid is moved and the Space-Conservation Law is applied (equations for moving control volumes solved)

LTD: Local Time Derivative approach: equations for fixed control volumes are solved, old solutions interpolated to the new cell-center location

Both approaches are OK...

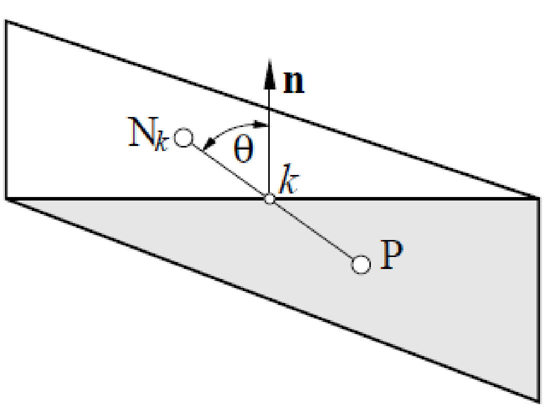
From PhD Thesis by Hidajet Hadžić, TUHH, 2005

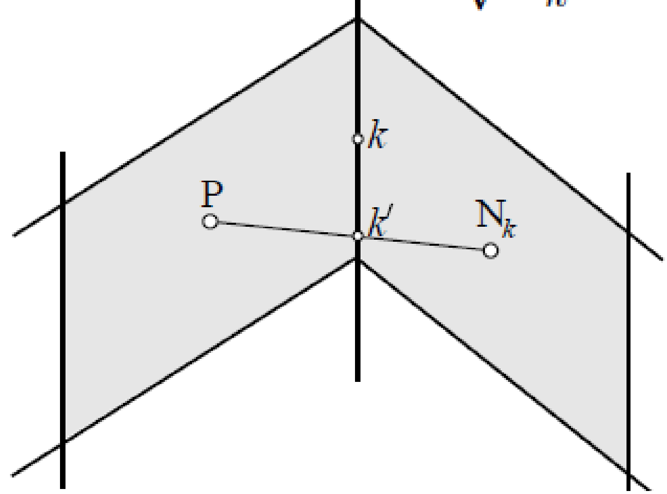
Grid Quality and Optimization – I

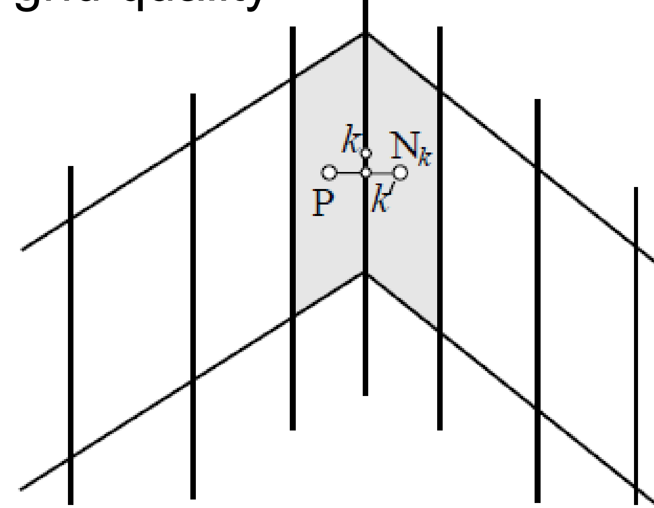
- Grid quality has a large effect on both stability and accuracy…
- Grid-non-orthogonality is one of the most important features; it should be minimized in optimization.
- The angle between face normal and connection between neighbor CV-centers is what matters…
- It is also desirable that the line connecting CV-centers passes through the face centroid.

$$\frac{|\mathbf{r}_k - \mathbf{r}_{k'}|}{\sqrt{S_k}}$$

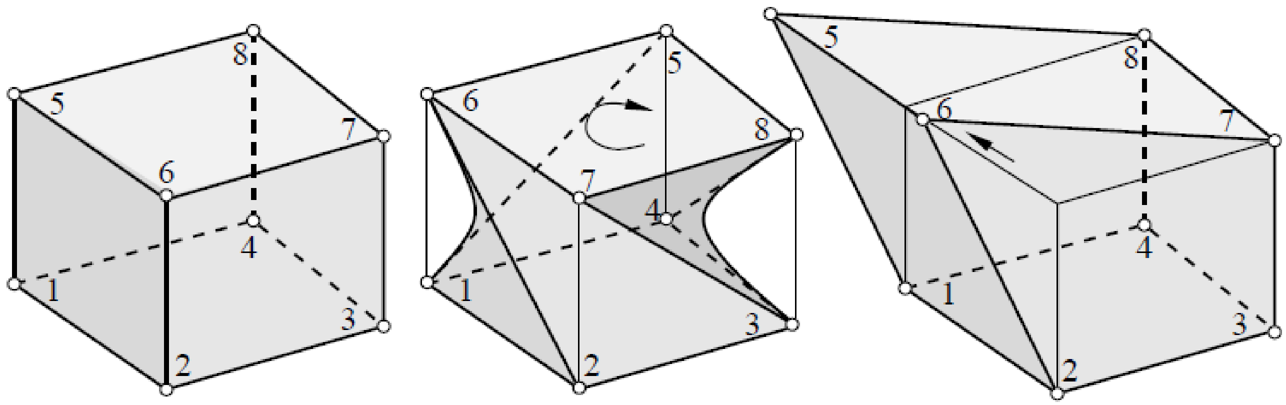
A measure of grid quality







Grid Quality and Optimization – II



Examples of poor grid quality:

- Warpage;
- Skewness;
- Flat tetrahedra;
- Prism layer wrapped around corner;
- Thin cells at a sliding interface…